Three Essays in Monetary Economics - Liquidity and its Effects on Inflation and Interest Rates

DISSERTATION
of the University of St. Gallen,
School of Management,
Economics, Law, Social Sciences
and International Affairs

to obtain the title of
Doctor of Philosophy in Economics and Finance

submitted by

Barbara Caroline Sutter
from
Ormalingen (Basel-Land), Winterthur (Zurich)

Approved on the application of

Prof. Paul Söderlind, PhD

and

Prof. Dr. Peter Kugler

Dissertation no. 4201
PublishingCenter Swiss National Bank
Zurich, 2013
The University of St. Gallen, School of Management, Economics, Law, Social Sciences and International Affairs hereby consents to the printing of the present dissertation, without hereby expressing any opinion on the views herein expressed.

St. Gallen, March 4, 2013

The President:

Prof. Dr. Thomas Bieger
**Acknowledgements**

I would like to thank my parents Erika and Georg Sutter and Jonas Stahel for their abiding support. Moreover, I thank Otto Huber for his continuing encouragement.

I owe special gratitude to Marcel Savioz for his constant support and for making it possible to combine work with my PhD studies. Moreover, I thank Karl Hug and Martin Schlegel for their support.

My special thanks are also addressed to my thesis supervisor Paul Söderlind for supporting me with helpful advice and comments. Moreover, I thank my co-authors Signe Krogstrup, Samuel Reynard, and Nikola Mirkov for the interesting and productive teamwork. Finally, I would like to thank my colleagues at the Swiss National Bank and at the University of St. Gallen as well as participants at the various seminars for fruitful discussions and helpful comments.
Contents

Summary 1

Zusammenfassung 3

1 Money and Inflation at Different Frequencies 5
   1 Introduction ................................................. 6
   2 Literature Review ........................................... 8
   3 Data .......................................................... 13
      3.1 Stationarity and Trend Removal .......................... 13
      3.2 Computation of the Frequency Components ............... 14
      3.3 Selection of the Frequency Bands ........................ 15
   4 Graphical Analysis ........................................... 16
      4.1 Spectral Analysis of Money Growth and Inflation ....... 17
      4.2 Frequency Components of Money Growth and Inflation .... 19
      4.3 Interpretation of the Graphical Analysis ................ 22
   5 Empirical Model .............................................. 24
   6 Regression Analysis of Money and Inflation .................. 30
      6.1 Frequency Dependence in the Relationship ................. 30
      6.2 Time-Variance in the Relationship ........................ 35
   7 Conclusion .................................................... 39
   A Some Properties of Bandpass Filters .......................... 41
   B Bandpass Filter Gain ........................................ 41
   C End-of-Sample Properties of Bandpass Filter ................ 43
   D Velocity Adjustment of Money ................................ 44
## 2 Liquidity Effects of Quantitative Easing on Long-Term Interest Rates

1 Introduction .......................................................... 48

2 How Do Central Bank Asset Purchases Affect Interest Rates? ........ 49

3 An empirical assessment ........................................... 52

  3.1 Identifying liquidity effects .................................. 53

  3.2 Methodology and Data ........................................ 54

  3.3 Results .......................................................... 56

  3.4 Robustness ....................................................... 58

4 Conclusion .................................................................. 61

A Figures ..................................................................... 63

B Tables ..................................................................... 68

C The Data ................................................................... 74

## 3 Central Bank Reserves and the Yield Curve at the ZLB

1 Introduction .................................................................. 78

2 The Different Effects of QE on Interest Rates ................. 80

3 Data .......................................................................... 82

  3.1 US Data ............................................................ 82

  3.2 Swiss Data ........................................................ 83

  3.3 Simple Data Inspection ......................................... 84

4 The Model ................................................................... 85

  4.1 General Setting and State Dynamics ......................... 85

  4.2 Short Rate and Bond Prices ................................... 86

5 Estimation ................................................................... 88

  5.1 Likelihood Function ................................................ 88

  5.2 Econometric Identification ...................................... 89

  5.3 Bayesian Inference ............................................... 90

6 Results ....................................................................... 94

  6.1 Model Performance ............................................... 94

  6.2 Parameters .......................................................... 97

  6.3 The Estimated Effect of Reserves on Interest Rates ....... 98

  6.4 Disentangling the Different Effects ......................... 101
7 Conclusion ........................................ 102
A Tables and Figures ................................. 104

References ............................................. 119
List of Figures

1 Money and Inflation at Different Frequencies 5
   1.1 Evolution of US Monetary Aggregates ....................... 7
   1.2 US Data ............................................. 14
   1.3 The Spectra of Inflation, Money Growth, and Output Growth 17
   1.4 Spectral Measures of Money Growth and Inflation .............. 18
   1.5 Frequency Components of Money Growth and Inflation .......... 20
   1.6 Money Leading Inflation by 2 Years .......................... 23
   1.7 Lowpass-Filtered Output Gap and Output Growth ............... 28
   1.8 The Gain of the BP-Filter at Each Observation ................. 42
   1.9 Leakage of the Bandpass Filter ............................ 44

2 Liquidity Effects of Quantitative Easing on Long-Term Interest Rates 47
   2.1 Non-Borrowed Reserves and Long Term Yields at the ZLB ........ 63
   2.2 In- and Out-of-Sample Fitted Values for 10-Year Yield ........ 64
   2.3 Dependent Variables ..................................... 65
   2.4 Regressors - Part I ..................................... 66
   2.5 Regressors - Part II .................................... 67

3 Central Bank Reserves and the Yield Curve at the ZLB 77
   3.1 Interest Rates and Central Bank Reserves in the US .......... 111
   3.2 Interest Rates and Central Bank Reserves in Switzerland .... 112
   3.3 Estimated Posteriors for the US ............................. 113
   3.4 Latent Factors for US Data ................................. 114
List of Tables

1 Money and Inflation at Different Frequencies .......................... 5
   1.1 Previous Findings on the Timing of the Relationship between Money and
       Inflation .......................................................... 11
   1.2 Stationarity Tests ................................................. 14
   1.3 Regression Results ................................................. 31
   1.4 Correlation of Frequency Bands .................................... 35
   1.5 Dummy Regressions ................................................. 38

2 Liquidity Effects of Quantitative Easing on Long-Term Interest Rates ... 47
   2.1 Pre-Crisis Regression Results ..................................... 68
   2.2 Regression Results including non-borrowed Reserves .............. 69
   2.3 Contributions to the Change in Yield ............................ 70
   2.4 Robustness Check I: Real Yield and Term Spread ................. 71
   2.5 Robustness Check II: 5-Year Treasury Yield ..................... 72
   2.6 Robustness Check III: Announcement Effects ........................ 73
   2.7 Events with Potential Announcement Effects ....................... 76

3 Central Bank Reserves and the Yield Curve at the ZLB .................. 77
   3.1 Sample Correlations .............................................. 104
   3.2 Parameter Estimates for the US ................................... 105
   3.3 Parameter Estimates for Switzerland ............................. 106
   3.4 Pricing Errors ..................................................... 107
   3.5 Variance Decomposition .......................................... 108
3.6 Factor Loadings for the US ........................................ 109
3.7 Factor Loadings for Switzerland ................................. 110
Summary

This thesis consists of three empirical essays in the field of monetary economics. Each part contributes to the analysis of how liquidity affects the economy.

The first paper studies the correlation between money and inflation. The main conclusion drawn from the literature is that money and inflation correlate only in the long run. There is very little precision as to the actual timing of the comovement. The analysis of US data suggests that not only very low frequencies but also frequencies typically ascribed to the business cycle are relevant for the comovement between money and inflation. Moreover, it shows that the comovement shifts across frequencies over time. This can explain the failure of linear models to detect the comovement in the time domain.

The second paper proposes that the traditional liquidity effect as presented in Friedman (1968) has changed in the recent years. The analysis gives evidence that the expansion of liquidity conducted by the Federal Reserve has affected long-term yields when the short-term yields came close to the zero lower bound. Consistent with the existing literature, the analysis successfully disentangles the supply effect from the liquidity effect.

The third paper proceeds in two steps. It first estimates the total effect of quantitative easing on term premia. In a second step it attempts to disentangle the supply from the liquidity effect. To estimate the effect of quantitative easing on the yield curve, central bank reserves are added as a fourth factor to an affine term structure model. The analysis is run for both US and Swiss data. The difference in the results provide information about the relative size of the supply and the liquidity effect at different maturities.
Zusammenfassung

Diese Dissertation besteht aus drei empirischen Analysen im Bereich der monetären Ökonomie. Jedes Kapitel liefert einen Beitrag zur Untersuchung, wie sich eine Veränderung der Liquidität auf die Wirtschaft auswirkt.


Der zweite Aufsatz stellt die These auf, dass sich der traditionelle Liquiditätseffekt analog zu Friedman (1968) in den letzten Jahren verändert hat. Die Analyse liefert Evidenz dafür, dass die Federal Reserve mit der Ausweitung der Liquidität die langfristigen Zinsen beeinflusst hat, als die kurzfristigen Zinsen nahe an die natürliche Untergrenze bei Null gestossen sind. Konsistent mit der existierenden Literatur differenziert die Analyse zwischen dem Angebots- und dem Liquiditätseffekt.

Der dritte Aufsatz liefert Schätzungen für den gesamten Effekt der ausserordentlichen geldpolitischen Massnahmen für US und Schweizer Daten. Hierzu werden Giorguthaben bei der Zentralbank als Mass von Liquidität als vierter Faktor einem Zinsstrukturmodell
Chapter 1

Money and Inflation at Different Frequencies

Abstract

Money and inflation are most often shown to correlate - if at all - only in the “long run”. There is very little precision with respect to the actual timing of the comovement, however. This chapter studies the correlation between money and inflation at different frequency bands. There are three main findings: First, the relation between money and inflation is frequency dependent, i.e. it changes across frequency bands. Second, a considerable time lag obscures the relation between money and inflation. Third, both the size of this lag and the most relevant frequency bands vary over time. Such time variability can explain the failure of linear models to show the comovement between money and inflation. The analysis suggests that not only very low frequencies but also frequencies typically ascribed to the business cycle are relevant for the comovement between money and inflation. The results are based on US data from 1960 to 2007 and the conclusions are drawn both from a graphical analysis of the variables’ frequency components and from regressions of headline inflation run on frequency components of money, output and interest rates.
1 Introduction

Does money matter for inflation? And if so, at what horizon is the comovement most pronounced? While the literature is very ambiguous in answering the first question, it is very vague on the second. From a policy perspective, however, both questions are highly relevant and may become of particular importance in the coming years. Quantitative easing has inflated reserves banks hold with the Fed to unprecedented levels. At the same time, short-term interest rates have been lowered to essentially zero. In the weak economic environment, however, growth rates of broader monetary aggregates have remained low, as the money multiplier dropped and has stayed on a low level since. If the monetary base is transmitted into broader monetary aggregates, the recovery of the money multiplier may result in severe inflationary pressure, unless appropriate counteractive measures are taken. To be able to manage broader monetary aggregates, the Fed introduced the payment of interest on reserves (IOR) in late 2008. When the economy will enter the phase of rebound, the Fed can raise IOR and thereby make it less attractive for banks to issue loans. However, the difficulty of correct timing remains unsolved. In order to have a better understanding of the future inflationary threat at present, there is a need for further investigation about the relationship between money growth rates and inflation.

Figure 1.1 shows the evolution of the monetary aggregates for the US. The upper panel shows that M1 and M2 have reached yearly growth rates of 20% and 10%, respectively. The lower panel depicts 10-year rolling averages of the respective yearly money growth rates. While the growth rate of M1 is at its historical high, M2 has not yet reached the extraordinary growth rates of the 1970s and early 1980s. Estimates of M3 growth rates remain well below 5%, but all three monetary aggregates show a clear tendency of increasing growth rates. Thus, the data suggests that monetary growth rates are getting close to their levels during the 1970’s which resulted in an extended inflationary period.

The relation between inflation and money postulated by the quantity theory of money has proven hard to show in the data of countries with low and stable inflation rates. Reasons are

\footnote{From John William’s Shadow government statistics.}
the instability of money demand, financial innovation, and other changes and shocks to the velocity of money circulation. Since velocity is unobservable, such movements obscure the relationship between the variables in the quantity equation. As Beck and Wieland (2007); Beck and Wieland (2008); Beck and Wieland (2009) show, however, ignoring money is especially dramatic when there is a large estimation error in other variables (potential output and equilibrium interest rates), which is likely to be the case in times of macroeconomic disequilibrium. Even if inflation is driven by real activity only in the short-run, it might still be advisable for a central bank to also focus on monetary aggregates if these have a measurable effect on inflation in the long run, as Gerlach (2003) notes. Hence, we need to have a more precise idea about the relationship between money and inflation.
If there is one clear-cut conclusion to draw from the literature, it is the one stating that
the relationship between money and inflation, if there is any, is strongest in the long run.
But there is very little precision as to the actual timing of the comovement. From a policy
perspective, however, it is important to have a better understanding of the timing of such
a comovement. Therefore, this paper studies the correlation between money and inflation
at different frequency bands. It is found that there is a strong comovement between money
and inflation not only in the long run, but also at frequencies that correspond to 3 to 8
years. Moreover, the results clearly indicate that the relationship is not stable over time. It
moves both with respect to frequencies as well as to the lag structure. While linear models
may suffer from biased estimates due to the absence of time stability, spectral techniques as
applied below seem to be a suitable instrument to further investigate this relationship. In
addition, the analysis shows that to identify the comovement between money and inflation,
different frequency bands should be taken into account.

The remainder of this paper is organized as follows. Section 2 gives an overview of the
relevant literature. Section 3 presents the data. The graphical analysis in Section 4
motivates the structure of the regression analysis. The empirical model is presented in 5,
followed by the results in Section 6. Section 7 concludes.

2 Literature Review

This section gives an overview of the literature on the relationship between money and
inflation. Due to the amplitude of the literature in this field, it is only a selective outline
with a focus on the different methods applied and the historic evolution of the findings. A
table tries to summarize what the literature has found with respect to the timing of the
correlation between money and inflation.

The empirical literature is most successful in showing the relationship between money and
inflation in the long run. Moreover, many cross-country studies have been able to show a
close relationship between money and inflation. Single-country studies were mainly suc-
cessful in confirming the relationship in the 1980's. At the time, the predominant approach
was to focus on the low-frequency relationship, estimating band spectrum regressions as proposed by Engle (1974, 1978). Lucas (1980) used spectral methods to show a one-for-one relation between money growth and inflation at frequency zero, i.e. in the very long run. The approach was criticized by McCallum (1984) as well as Whiteman (1984). Whiteman (1984) argues that the slope in Lucas’ scatter plots can be approximated by the sum of distributed lag coefficients computed using the spectral density implied by a state space representation. Taking up this idea, Sargent and Surico (2008, 2009) show that in a dynamic stochastic general equilibrium environment, Lucas’ one-for-one relationship is destroyed when the monetary authority reacts sufficiently aggressively to inflationary pressures. This finding is in line with the evidence from panel studies showing that the strong relationship is typically driven by hyper-inflationary countries (see e.g. De Grauwe and Polan (2005) or Dwyer and Hafer (1988)).

The predominant view in the empirical literature of the 1990’s was that the strong relationship had disappeared. These more recent studies applied a different approach, testing explicitly for coefficient restrictions implied by the quantity theory in vector autoregression models. Examples are Geweke (1986), and King and Watson (1997). Moreover, Granger-causality tests were performed in cointegrating frameworks. Techniques in the frequency domain had subsided. Kirchgässner (1985) provides a literature review of Granger causality between money and output: although the literature is ample in the field, the evidence of causality is highly ambiguous.

Studies that are successful in showing a positive relation between money and inflation over time feature two components: First, they find that there needs to be some kind of adjustment to changes in velocity which is, for example, essential for the findings in Reynard (2006, 2007). Second, they combine the quantity equation with the Phillips curve. Phillips (1958) showed that wage inflation rates in the UK are well explained by the level and the rate of change of unemployment. The ‘traditional’ Phillips curve therefore relates the wage inflation rate to the unemployment rate. Later, the output gap has been used as a measure of real activity. In the New Keynesian framework inflation is usually modeled such that it reacts to some measure of the output gap and expected inflation, and sometimes also to lagged inflation rates. Monetary aggregates have no effect on inflation.
in these models. Much of the DSGE literature has come to the conclusion that since the relationship between money and inflation holds only in the long run, there is no gain in analyzing monetary aggregates for shorter-term policy making. Woodford (2007, 2008) shows that New Keynesian models produce very similar inflation dynamics as found by Assenmacher-Wesche and Gerlach (2008a) without including any measure of money. He accuses empirical work in the monetarist tradition to often emphasize on simple correlations rather than on the estimation of structural models. He stresses the fact that results from reduced-form model estimations should be interpreted with caution.

\footnote{Woodford (2007) refers to an earlier version of this paper that appeared in 2006 as a CEPR discussion paper no. 5632.}
Table 1.1: Previous Findings on the Timing of the Relationship between Money and Inflation

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Data</th>
<th>Approximate Timing</th>
<th>Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friedman (1972)</td>
<td>US, 1963-1971</td>
<td>2 years</td>
<td>Maximum cross-correlation between money and inflation at 20-23 months for M1/M2.</td>
</tr>
<tr>
<td>Thoma (1994)</td>
<td>US, 1959-1989</td>
<td>1-2 years</td>
<td>Finds causality at frequencies corresponding to 1 to 2 years for M1 and M2.</td>
</tr>
<tr>
<td>Assenmacher-Wesche and Gerlach (2007b,a, 2008a,b)</td>
<td>UK, JP, US, EA, CH</td>
<td>&gt; 4 years</td>
<td>Causality at zero frequency; potential reverse causality below one year. Band spectrum regressions show a long-run relation between money growth and inflation.</td>
</tr>
<tr>
<td>Reynard (2007)</td>
<td>EA, US, CH</td>
<td>2-5 years</td>
<td>Velocity adjusted excess liquidity is shown to be strongly correlated with subsequent inflation.</td>
</tr>
<tr>
<td>Anderssen (2008)</td>
<td>Panel, 1971-2003</td>
<td>&gt;2 years</td>
<td>Performs panel band spectrum regressions (9 countries) and finds impact of excess money on inflation at the 2-4 year frequency and at low frequency.</td>
</tr>
<tr>
<td>Kaufmann and Kugler (2008)</td>
<td>EA, 1977-2006</td>
<td>&gt;4 years</td>
<td>A permanent change in money growth affects inflation permanently. The error decomposition shows that after 15 (40) quarters, money accounts for more than 30% (50%) of the variability of inflation.</td>
</tr>
</tbody>
</table>

Note: This table aims at collecting as much information from the literature as possible with respect to the exact timing of the comovement between money and inflation. This “Approximate Timing” derived is often not explicitly mentioned in the original paper and subject to interpretation.
Sims (2009) shows in a structural VAR analysis that the main mechanism underlying the data can hardly be the one of the New Keynesian Phillips curve. Lucas (2006) claims that New Keynesian models should be reformulated, arguing that “ignoring monetary information would entail ignoring the only explanation we have for the inflation of the 1970’s as well as ignoring the only principle that proved useful in bringing that inflation to an end.” Nelson (2008) shows in a standard New Keynesian analysis that the central bank can control long-run interest rates through inflation rates, and that its only instrument for determining inflation is the money growth rate. In the study of Favara and Giordani (2009), shocks to money have a substantial and persistent effect on output, prices, and interest rates. This finding is at odds with the New Keynesian models where output, prices, and interest rates can be determined without the influence of monetary aggregates. Canova and Menz (2009) study data for the US, the UK, Japan, and the euro area and find that money is important for output and inflation fluctuations, and that the contribution of money changes over time. Particularly, they show that models ignoring monetary aggregates reveal a distorted representation of the sources of the business cycle. Finally, as Fitzgerald (1999) puts it: “If there is a close relationship between money growth and inflation in the long run, then ignoring money growth in short-run policymaking poses a risk”.

Gerlach (2003) combines the quantity-theoretic relation between money and inflation with the Phillips curve. Assenmacher-Wesche and Gerlach (2007b,a, 2008a,b) show in the so-called two-pillar Phillips curve framework that inflation is driven by the output gap in the shorter and by money in the longer run. However, they refuse to get precise on how long the short and the long run are.

The literature is in general very ambiguous and imprecise when it comes to the horizon in which money and inflation are supposed to co-move. From the perspective of the policy maker, this is a very important question, however. Table 1.1 gives an incomplete overview of empirical studies that contribute to the literature on the timing of the comovement. It must be noted, however, that most of the papers covered in the table do not aim at defining this horizon but focus on the more fundamental question of the existence of the relationship.
3 Data

The analysis is performed for US data spanning from 1960 to 2007. The data used are the consumer price index (CPI), industrial production, the monetary aggregate M2, and the three-month Libor rate. Data on prices and monetary aggregates are retrieved from the FRED. The BIS provides data on industrial production, and the longest time series for interest rates can be found in the International Financial Statistics managed by the International Monetary Fund. Industrial production is available every month. Using industrial production, rather than data on GDP, hence allows to conduct the estimates on a monthly basis. The number of observations is crucial for spectral analysis because it considerably enhances the reliability of the estimates, especially at low-frequencies. Figure 1.2 depicts the raw data. Seasonal adjustment is in principal a symmetric filter and should not produce a phase shift. Moreover, it should only filter out high-frequency movements in variables but not change medium and low-frequency variability. Thus, there is no obvious reason why not to use seasonally adjusted variables. However, the spectral density of a seasonally adjusted variable exhibits shifts as compared to its non-adjusted counterpart, i.e. the spectral densities peak at different frequencies. For this reason, all data enters the analysis as unadjusted time series.

3.1 Stationarity and Trend Removal

A covariance stationary variable can be decomposed into an integral of periodic components. Moreover, Assenmacher-Wesche and Gerlach (2007b) argue that nonstationary series, by definition, do not experience permanent effects of shocks. Therefore, they cannot explain non-stationary series in the long run.

To test for covariance stationarity, Table 1.2 depicts the results of unit root tests for all variables. The tests are run on the month-on-month growth rates of the cpi, money, and GDP - denoted $\Delta p_t$, $\Delta m_t$, and $\Delta y_t$, respectively - and on the first difference of the three month libor $\Delta i_t$. The results clearly reject the null hypothesis that a unit root exist for all variables. Thus, trend removal prior to selecting frequency bands is not necessary.
Table 1.2: Stationarity Tests

<table>
<thead>
<tr>
<th>Sample</th>
<th>Augmented Dickey Fuller</th>
<th>Phillips-Perron</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t$-Stat</td>
<td>Prob$^\ddagger$</td>
</tr>
<tr>
<td>$\Delta p_t$ 1913/02 to 2011/04</td>
<td>-9.2346</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\Delta m_t$ 1959/02 to 2011/04</td>
<td>-4.2030</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\Delta y_t$ 1921/02 to 2011/04</td>
<td>-9.1863</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\Delta i_t$ 1960/02 to 2011/04</td>
<td>-18.2247</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

$^\ddagger$ MacKinnon (1996) one-sided p-values. $^\ddagger$ Automatic lag selection based on SIC (max = 22). $^\ddagger$ Newey-West using Bartlett kernel. A constant is included in the tests.

3.2 Computation of the Frequency Components

The frequency components are computed by bandpass filtering the monthly growth rates of the raw data. For the graphical analysis, a two sided filter is used. The regressions section reports results separately for one-sided and two-sided filters. Some of the main properties of bandpass filters are sketched in appendix A. To maximize the precision of the estimates, the filter is designed to use all available information at each point in time.
The series of inflation starts as early as 1913, M2 in 1921, interest rates date back to 1954 and industrial production is available as of 1959. All data end in June 2011. With a view to potentially poor end-of-sample properties of bandpass filters, the entire sample of each variable is used to compute the respective frequency components.\(^3\) In principle, the optimal bandpass filter is symmetric and therefore does not cause a phase shift. The two-sided filter employed here, because it is only an approximation to the optimal filter, uses relatively more information from the future at the beginning of the sample and relatively more past information towards the end of the sample. This causes the resulting frequency components to exhibit a phase shift that evolves dynamically over time. Conversely, the one-sided filter does not consider future observations.

3.3 Selection of the Frequency Bands

The main question of this paper is at which frequencies money growth and inflation exhibit the highest correlation. To this end, the time series need to be decomposed into their respective frequency components. The bandpass filter is a natural way to achieve this. As a first step, however, the frequency bands must be defined. From the economic point of view, there are at least three frequency bands to be distinguished: seasonality, business-cycle fluctuations, and long-run movements. The frequencies usually attributed to the business-cycle are those corresponding to fluctuations with a duration between 1.5 and 8 years, as suggested by Baxter and King (1999). As a consequence of this definition, long-term movements could be attributed to frequencies corresponding to more than 8 years, and seasonal fluctuations to less than 1.5 years. In the literature, long-term movements are often ascribed to zero-frequency, e.g. by Lucas (1980) and more recently by Benati (2009) and Sargent and Surico (2009, 2008). In applications of band spectral regressions following Engle (1974, 1978), Assenmacher-Wesche and Gerlach (2007b,a, 2008a,b) define the low frequency as corresponding to more than 4 years. Fitzgerald (1999) uses 2-, 4-, and 8-year moving averages to describe the long-run relation between money and inflation. From the point of view of the monetary authority, a straightforward threshold would be the frequency corresponding to three years, as policy decisions are typically based on a forecast horizon of this duration. Moreover, it may be interesting to further divide up the

\(^3\)See Appendix C for more details on the end-of-sample properties of the bandpass filter.
relatively wide business-cycle component.

There are some technical difficulties that must be taken into account when computing the frequency components. Bandpass filters are imprecise, i.e. there is leakage in empirical approximations of filters to their ideal form. Leakage means that movements of a variable are passed on by the filter even though they stem from frequencies just outside of the passband. Appendix B describes the phenomenon of leakage in more detail. Reliability can be enhanced in two ways: either by widening the passbands or by dropping certain frequencies in between two passbands. The former solution makes the filter more precise. The second solution avoids that two different frequency-band components are driven by the same frequencies. These technical difficulties imply one major restriction on the frequency band selection: there can only be very few different bands at low frequency. Figure 1.9 in appendix B suggests that at low frequency, e.g. corresponding to fluctuations of more than 8 years, the number of frequency bands should not exceed two. At higher frequencies, the leakage is relatively small.

The combination of these three aspects of the band selection leads to the following choice of frequency bands. First, the band corresponding to 0 to 1.5 years captures seasonal fluctuations. Those fluctuations especially interesting for monetary policy correspond to frequencies between 1.5 and 3 years and are therefore ascribed to the second band. The third and fourth band take into account different movements of the business cycle: one corresponds to fluctuations of 3 to 5 years, the other of 5 to 8 years. Finally, the low-frequency component captures movements with a duration that exceeds 8 years.

4 Graphical Analysis

Previous studies, such as Lucas (1980), Fitzgerald (1999), or Assenmacher-Wesche and Gerlach (2008a), have used graphical analysis to show the comovement in money and inflation. In this vain, this section uses graphical techniques to shed some more light on the frequency dependence of this comovement. In a first step, spectral techniques are employed to answer the question whether there is any comovement between money and
inflation other than in the very long run, i.e. away from the zero frequency. In a second step, this section looks at money growth and inflation within different frequency bands and gives evidence that money might lead inflation by several years.

4.1 Spectral Analysis of Money Growth and Inflation

The spectrum of a time series shows the fraction of the total variance that is associated with the respective frequency. Figure 1.3 shows the spectra for inflation, money growth, and output growth. The frequency \( \omega \) relates to the time period as \( j = 2\pi / \omega \). Thus, with monthly data \( \omega = 0.065 \) corresponds to a period of approximately eight years and \( \omega = 0.35 \) to one and a half years. While it is true for both inflation and money growth that the low frequencies contributes most to the variables’ variances, this is not the case for output growth. The spectra indicate that money and inflation exhibit a similar pattern in the frequency domain. To look deeper into this relationship, Figure 1.4 shows further spectral measures of money and inflation.

Figure 1.3: The Spectra of Inflation, Money Growth, and Output Growth

The spectra are computed using data from Feb-1959 to Jun-2011. Smoothing is conducted with a Parzen window of the size \( M = T/5 \).
The cross spectrum in the upper left panel indicates that money and inflation exhibit the closest relation at low frequencies. Since the computation of these spectral measures always hinges on some sort of smoothing, it is very difficult to tell the exact frequency bands from these graphs. But since \( \omega = 0.05 \) corresponds to a period of more than 10 years, the most elevated part of the curve can be attributed to the long run. However, there is another increase in the cross spectrum around the frequency corresponding to 5 years. Hence, this could indicate that money and inflation comove not only in the long run but also within business cycle frequencies. The coherence, depicted in the upper right panel, provides even stronger evidence for this claim. While the cross spectrum indicates the fraction of the total covariance that is associated with each frequency, the coherence is an arguably more informative measure. As Granger (1969) puts it, the coherence is essentially the square of the correlation coefficient between corresponding frequency components of two variables. Technically speaking, the coherence normalizes the cross spectrum by the spectra of the

**Figure 1.4:** Spectral Measures of Money Growth and Inflation

These spectral measures are computed using data from Feb-1959 to Jun-2011. Smoothing is conducted with a Parzen window of the size \( M = T/5 \).
two variables. This is why the peaks away from the zero frequency are comparatively larger than in the cross spectrum. Measured by the coherence, the comovement seems to be just as large at frequencies corresponding to two years ($\omega = 0.26$) and 1 year ($\omega = 0.52$) as at the lower frequencies.

The concept of the co-spectrum accommodates relationships between components which are out of phase. The co-spectrum in the lower left panel isolates the covariances that are in phase, i.e. contemporaneous comovements. Conversely, the quadrature spectrum in the lower right panel measures the covariances that are out of phase, i.e. it allows for a lag. The comparison of the two measures suggests that at low frequencies corresponding to more than 10 years, there is considerable comovement between money and inflation both in phase and out of phase. At the frequency around $\omega = 0.1$ or 5 years, the comovement out of phase is almost of the same magnitude as the first peak at lower frequency, while in phase it is much smaller. This is a first indication that it may be necessary to look for comovements between money and inflation not only at different frequencies but also at different lag structures.

Spectral analysis of money and inflation suggests that there is a strong contemporaneous comovement between inflation and M2 growth. It also indicates that there is considerable comovement at higher frequencies, corresponding to 3 to 5 years, but that this relationship may be hidden by time lags. The next subsection separates the different frequency components of the two variables to gain more precise insights into the comovement at different frequencies.

4.2 Frequency Components of Money Growth and Inflation

To study whether the comovement between money growth and inflation depends on the frequency, this section provides a graphical analysis of the two variables for different frequency bands. The frequency components are computed by bandpass filtering the respective time series. This section employs a two-sided bandpass filter as described in Section 3 and in appendix A. Since both variables are filtered, the phase shift caused by the approximation
of the filter to its optimal counterpart applies to both variables and should therefore not cause any problems for interpretation. Figure 1.5 presents the frequency components of inflation and the growth rate of M2 for different frequency bands. As described above, the

**Figure 1.5:** Frequency Components of Money Growth and Inflation

These graphs show the frequency components of the variables as indicated in the legend. The asterisk * indicates that the original time series has been filtered. The frequency components are computed using a bandpass filter on the entire range of data available for each variable. The data on inflation ranges from Jan-1913 to Jun-2011; data on M2 from Jan-1921 to Jun-2011.

The frequency bands considered correspond to fluctuations that correspond to 1.5 to 3 years (depicted in the upper left panel), to 3 to 5 years (in the upper right panel), to 5 to 8 years (in the lower left panel), and to more than 8 years to describe the long run (in the lower right panel). The graph for high frequencies corresponding to less than 1.5 years is omitted because the seasonality pattern is not the purpose of this paper.

It is difficult to detect any comovement in the upper left graph representing cycles of 1.5
to 3 years that reflects the horizon of monetary policy. The simple correlation amounts to -0.4. Money growth exhibits a much larger amplitude than inflation which means that money growth fluctuates more than inflation within this frequency band. During the 1980s, the amplitude of inflations increases and seems to exhibit a similar shape to that of money, but shifted in time. At frequencies corresponding to 3 to 8 years, represented in the upper right and the lower left panels, the relation between money and inflation is more easily visible. However, there also seems to be a time shift between the two variables. For the band corresponding to 3 to 5 years, the simple correlation is -0.55 and even -0.75 for the band from 5 to 8 years. The lower right panel shows money growth and inflation at their lowest frequency, i.e. their respective long-run movement. The pattern here is not entirely clear. It looks like the correlation, where there seems to be correlation, is contemporaneous.\footnote{It must be noted here that low-frequency components are most sensitive to both the sample and the respective filter selected to compute the components. While the frequency components do not differ much when a different filter is used, e.g. the Christiano-Fitzgerald filter, the low-frequency component does differ considerably.}

Overall, there seems to be a time shift between some frequency components of money and inflation.\footnote{In what follows, it could of course be argued that inflation leads money. The comovement may be just as close or even closer when inflation is shifted forward in the graphs. This graphical analysis focuses on correlations only and does not claim to give evidence of any causal relations.} This is not a new finding and has been described in the literature. King and Watson (1996) analyze the main nominal and real variables of the US in the frequency domain. To get an idea of the interaction between the variables, they calculate correlations of the business-cycle frequency components. They also find that there are lags in the relationship between money and output as well as between output and prices during certain periods.\footnote{In particular, King and Watson find that prices and output are more strongly correlated at longer lags. At a lag of ten quarters, the correlation amounts to 0.45. They interpret this lag as price stickiness in response to nominal disturbances. Graphical representation of output and inflation as presented in this section for money and inflation does not suggest such a lagged relationship, however. At frequencies corresponding to less than 3 years, the correlation is largest with output growth lagged by only one month. At frequencies corresponding to 3 to 8 years, the correlation is largest with output lagged between a half year and a year.} Assenmacher-Wesche and Gerlach (2008b) find that Swiss money growth leads inflation by approximately 3 years at frequencies corresponding to more than 8 years. Such time shifts are often attributed to gradual changes in velocity that are fairly difficult to measure.
Simple correlations suggest that at frequencies between 1.5 and 8 years, the comovement is largest when money leads inflation between 1.5 and 4 years. Therefore, Figure 1.6 shows money with a lag of 2 years for each frequency band. From the beginning of the sample up to the end of the 1970s, the correlation seems to be largest at business cycle frequencies corresponding to 3 to 8 years, visualized in the upper right and lower left panel of Figure 1.6. Towards the end of the 1970s, the lag of two years seems to be too short. The correlation is restricted to the horizon of 5 to 8 years, at a lag of approximately 3 year, during the end of the 1970s until the mid 1980s. From the mid 1980s to the mid 1990s, there seems to be no correlation in fluctuations of more than 3 years. During this period, however, the frequency band of 1.5 to 3 years exhibits some correlation between the two variables. Although the amplitude is considerably larger for money growth, the two move quite close together. After 1995, the correlation shifts back to the frequency corresponding to 3 to 5 years, and also to 5 to 8 years, but at a slightly different lag. At the higher frequencies, the correlation disappears again.

Most of the time the correlation between money and inflation concentrates at frequencies corresponding to 3 to 8 years, with a lag of 2 to 3 years. The exception is the period between 1985 to 1995 where the comovement shifts to higher frequencies.

4.3 Interpretation of the Graphical Analysis

There are three main insights from the graphical analysis above: First, the correlation between money and inflation seems to be strong not only at the lowest frequencies but also at frequencies that correspond to the business cycle. Second, the comovement between money and inflation might not always occur at the same frequency. Finally, there seems to be a time shift of 2 to 4 years that must be taken account of.

To put these findings into perspective: the literature often suggests that the link between money and inflation is observable only in high-inflation environments. With the Volcker era starting in 1979, the empirical relationship is typically found to disappear from the data. Moreover, the link between money and inflation is typically found only in the very long run, or at zero frequency. The Figures 1.5 and 1.6 show that most of the time, the
These graphs show the frequency components of the variables as indicated in the legend. The frequency components are computed using a bandpass filter on the entire range of data available for each variable. The data on inflation ranges from Jan-1913 to Jun-2011; data on M2 from Jan-1921 to Jun-2011.

closest comovement between money and inflation occurs at frequencies corresponding to 3 to 8 years. With the Volcker disinflation period between 1980 and 1984 and the subsequent years, this correlation disappears. Interestingly, this is exactly the period when the correlation appears in the higher frequency band corresponding to 1.5 to 3 years. This suggests that, opposed to what is typically concluded in the literature, the relationship did not disappear at the time, but might have moved to a different frequency band. Moreover, the graphs suggest that a contemporaneous analysis will find a comovement only at the lowest frequencies and not at what corresponds to the business cycle.

What could shift the correlation between money and inflation to the shorter horizon? In 1979 Paul Volcker took over the Fed. He decidedly wanted to bring down inflation, running a very restrictive monetary policy. Goodfriend and King (2005) argue that in the first years
of the 1980s, Volcker’s policy was lacking credibility which showed in persistently high long-term interest rates. While both inflation and the short-term interest rates reached reasonably low levels by the year 1983, 10-year Treasury bonds persisted above the level of 10% for two more years. Indeed, it took inflation several years to react, while money growth rates were reduced already in the late 1970s. If the continuously restrictive path of Volcker’s policy would successively reinforce the intentions of the Fed, and thereby successively enhance credibility, and if money growth mirrors the restrictive policy, then we would expect inflation to react to money at higher frequencies during and shortly after the Volcker era. Moreover, the elevated long-term interest rates reflected uncertainty about future inflation. High expected inflation induces contracts to adapt faster to changes in expectations. Hence, inflation reacts faster to changes in the money growth rate when inflation is high. Moreover, the steady increase in credibility of Volcker’s policy may have increased the significance of the money growth rate as a predictor for inflation. Then, inflation expectations and thus contracts adjusted quicker to changes in the money growth rate. After some time, the money growth rate lost its predictive power over inflation again with the literature at the time that showed a vanishing correlation between money and inflation. This way, the comovement may have shifted back to lower frequencies.

5 Empirical Model

While the graphical analysis gives a hint at correlations within different frequency bands, it is headline inflation that is of interest for monetary policy. Therefore, the model is set up to empirically test which frequency components of money are most important for headline inflation. Money needs to be decomposed into different frequency components. Then, inflation can be regressed on these frequency components to estimate the contribution of money to headline inflation at the different frequencies.

The quantity equation forms the starting point of the analysis. Formally, after taking logs and first differences, it reads

$$\Delta p_t = \Delta m_t - \Delta y_t + \Delta v_t.$$  (1.1)
Empirical analyses of the quantity equation are thought to fail because of fluctuations and shifts in velocity that cannot be directly measured. Therefore, the bulk of the empirical approaches test the model in (1.1) in the long-run only, assuming constant velocity to be a good approximation in the long run. A straightforward way to estimate the quantity equation at low frequency is Engle’s band spectral approach. What Engle (1974, 1978) suggests comes down to pre-multiplying both sides of (1.2) with a low-pass filter, which does not corrupt the error terms and therefore does not affect inference. The band-spectral approach allows to directly compare the two frequency bands, the low-pass band and it’s respective high-pass band, that result from defining some threshold frequency. It also allows to compare the results from defining different thresholds. But the low-frequency component contains all frequencies below the threshold, i.e. including zero frequency. Thus, the band spectral approach does not allow to look at business cycle frequencies separately. Moreover, as both sides of the equation are pre-multiplied by some filter, the band spectral approach allows only to estimate the correlation between variables within a certain frequency band.

Instead, the approach here is to explain headline inflation with the different frequency components of money and output growth and velocity. Suppose the relation between two variables \( y_t \) and \( x_t \) can be stated in a simple distributed-lag model of the form

\[
y_t = \beta(L)x_t + \varepsilon_t \quad \text{with} \quad \beta(L) = \sum_j \beta_j L^j
\]  

(1.2)

where \( L \) denotes the lag operator. To find out at which frequency band the explanatory power of \( x_t \) is largest, \( x_t \) needs to be decomposed into different frequency bands. Let \( a(L) \) denote the distributed lags of a two-sided perfect low-pass filter.\(^7\) Suppose we restrict \( \beta(L) \) in model (1.2) to be frequency dependent, such that the explanatory power of variable \( x_t \) over \( y_t \), and hence \( \beta(L) \), differs for low- and high-frequency components of \( x_t \). Let \( \tilde{x}_1^t \equiv a(L)x_t \) and \( \tilde{x}_2^t \equiv (1 - a(L))x_t \) denote the low- and high-frequency components of \( x_t \), respectively. To impose frequency dependence on the coefficients, \( \beta(L) \) is restricted to be

\(^7\)Appendix A provides some basic facts on bandpass filters.
of the form
\[ \beta(L) = \tilde{\beta}^1(L) \cdot a(L) + \tilde{\beta}^2(L) \cdot (1 - a(L)). \] (1.3)

The dimension of \( \tilde{\beta}^1(L) \) is further restricted to be one: some regressions are run contempo-
ranously, i.e. with the restriction \( \tilde{\beta}^f(L) = \tilde{\beta}^f L^0 \), and others assume a lag of 2 or 3 years,
where the restriction becomes \( \tilde{\beta}^f(L) = \tilde{\beta}^f L^24 \) or \( \tilde{\beta}^f(L) = \tilde{\beta}^f L^{36} \), respectively. With these
restrictions and the lag structure truncated at \( M \), the distributed lag model in (1.2) can
be written as
\[ y_t = \tilde{\beta}^1 \sum_{j=0}^{M} \alpha(L) x_{t+j} + \tilde{\beta}^2 \sum_{j=0}^{M} [1 - \alpha(L)] x_{t} + \varepsilon_t. \] (1.4)

While in the distributed lag model in (1.2), \( M \) coefficients would need to be estimated,
there are only two coefficients to be estimated in model (1.4) as \( \alpha(L) \) and \( [1 - \alpha(L)] \) is
restricted to correspond to the corresponding bandpass filters.
The general model (1.2), with the imposed frequency dependence in (1.3), can be written as
\[ y_t = \tilde{\beta}^1(L) \tilde{x}_t^1 + \tilde{\beta}^2(L) \tilde{x}_t^2 + \varepsilon_t. \] (1.5)

For \( n \) frequency bands, the model in (1.2) can be written as
\[ y_t = \sum_{f=1}^{n} \tilde{\beta}^f(L) \tilde{x}_t^f + \varepsilon_t \] (1.6)
with
\[ \tilde{x}_t^f = a^f(L) x_t \quad \text{and} \quad \sum_{f=1}^{n} a^f(L) = \iota(t \times 1) \] (1.7)
where \( \iota(t \times 1) \) is an indicator vector of size \( t \). Condition (1.7) ensures that the frequency
components \( \tilde{x}_t \) add up to the original time series \( x_t \). Thus, equation (1.6) restricts \( \beta(L) \) in
equation (1.2) to satisfy \( \beta(L) = \sum_{f=1}^{n} \tilde{\beta}^f(L) a^f(L) \).

Applying this to the quantity equation in (1.1), the model to be estimated is a distributed
lag model of the form
\[ \Delta p_t = \sum_{f=1}^{n} a^f(L) \Delta m_t - \sum_{f=1}^{n} a^f(L) \Delta y_t + \sum_{f=1}^{n} a^f(L) \Delta v_t, \quad \text{with} \quad \sum_{f=1}^{n} a^f(L) = \iota(t \times 1). \] (1.8)
Again for notational ease, define \( \tilde{x}_t^f \equiv a^f(L)x_t \) and restrict \( \sum_{f=1}^{n} \tilde{x}_t^f = x_t \) so that the frequency components add up to the original variables for \( x_t = [\Delta m_t \ \Delta y_t \ \Delta i_t] \). The model to be estimated is then

\[
\Delta p_t = \sum_{f=1}^{n} \tilde{\beta}_{m}^f(L)\Delta \tilde{m}_t^f - \sum_{f=1}^{n} \tilde{\beta}_{y}^f(L)\Delta \tilde{y}_t^f + \sum_{f=1}^{n} \Delta \tilde{\beta}_v^f(L)\Delta \tilde{i}_t^f + \varepsilon_t. \tag{1.9}
\]

The strict one-for-one relation between money and inflation as postulated by the quantity equation would be straightforward to test: it would require that the bandpass filter weights of all leads and lags, multiplied by their respective frequency dependent coefficient estimate, would add up to zero; and the contemporaneous to unity. However, since the interest rate is probably an imprecise proxy for velocity, we cannot expect to find the unitary relationship of the quantity equation.

This model comprises several nice features: First, there is no loss of degrees of freedom (as in band spectral regressions) because the dependent variable is unfiltered.\(^8\) Second, it is very general and nests models such as the two-pillar Phillips curve. While the Phillips curve literature suggests a positive impact of output growth on inflation, the quantity equation postulates a negative correlation. In this model, each variable is allowed to impact headline inflation in a frequency dependent fashion, so it can take account of both relationships.

There are two major differences between equation (1.9) and the models typically applied within the two-pillar Phillips curve framework: the output (gap) and the treatment of velocity. The literature of the two-pillar Phillips curve follows the New Keynesian literature in adding a measure of the output gap to the empirical model in equation (1.8).\(^9\) Assenmacher-Wesche and Gerlach (2008b), for example, set up a model of headline inflation that explains inflation at low frequency with the quantity equation variables and at high frequency, inflation is driven by the output gap. The estimation results of such a

\(^8\)Since the dependent variable is unfiltered, there is no loss in degrees of freedom stemming from filtering. See also Assenmacher-Wesche and Gerlach (2008b) on this issue. If the frequency components are thought to be estimated with a measurement error, then a classical errors-in-variables problem could arise. In this case, an attenuation bias would tilt the coefficients towards zero. Therefore, the estimates could be interpreted as conservative.

model require special attention when the band spectral techniques along the lines of Engle (1974) are applied. A measure of potential output can be thought of as lowpass filtered output $c^{LF}(L)y_t$. Hence, the output gap is the corresponding highpass filtered output $(1 - L)c^{LF}(L)y_t$. Then, when the estimation is run for low frequencies, what remains is the portion of the output gap that corresponds to low frequencies only. If the output gap is thought to correspond to business cycle fluctuations, then there should be no long-run fluctuations in the output gap per definition, and the output gap filtered with a lowpass filter should be a constant. Hence, in a low-frequency regression, the output gap should be expected not to explain inflation when a constant is included. Figure 1.7 illustrates this issue. It shows the lowpass filtered output gap (dark solid line) and the lowpass filtered output growth (red dashed line). The output gap is supposed to explain high-frequency movements in inflation, and output growth should drive inflation in the long run. However, the graph shows that the lowpass filtered output gap is almost constant. Equation (1.8)

\begin{equation}
\text{Figure 1.7: Lowpass-Filtered Output Gap and Output Growth}
\end{equation}

allows to add different frequency components of output growth as explanatory variables. Since the output gap is nothing but output growth at certain frequencies, it allows to precisely measure the impact of the output gap, alongside with low-frequency movements of real growth, i.e. potential output, on inflation.

The second issue that deserves attention is the treatment of velocity in equation (1.8).
Velocity is not directly observable. It is typically thought that money demand changes with the interest rate, as interest rates determine the opportunity costs of holding money. To account for shifts in money demand, money supply must be adjusted because such demand-induced changes should not affect prices but rather cause a change in velocity. Reynard (2006, 2007) therefore suggests adjusting money growth with some measure of velocity. This adjustment comes down to estimating interest rate elasticity of money demand in a first step. The corrected money growth rate is then used to empirically test whether money contains information over inflation at higher frequencies. In particular, Reynard (2006, 2007) estimates interest rate elasticity of money demand in a first step from

\[ v_t \equiv m_t - p_t - y_t = c + \varepsilon \cdot \ln i_t^{10y}. \] (1.10)

Then, he runs regressions of the form

\[ \Delta p_t = c + \alpha_1 \Delta m_t - \alpha_2 \Delta y_t^* + \alpha_3 \Delta i_t^* + u_t \] (1.11)

where

\[ \Delta y_t^* = \Delta m_t - \Delta y_t^* + \varepsilon \Delta i_t^* \] (1.12)

with * denoting a HP-filtered variable, so that \( \Delta y_t^* \) is the potential growth rate, or the long-run growth rate, and \( i_t^* \) denotes some sort of equilibrium interest rate. So money growth in excess of potential output growth is adjusted by equilibrium velocity. The goal is to see whether money can explain inflation in the short term after imposing long-term restrictions.

The regression equation (1.20), skipping the lagged regressors, can be thus written as

\[ \Delta p_t = c + \alpha_1 \Delta m_t - \alpha_1 \Delta y_t^* + \alpha_1 \varepsilon \Delta i_t^* + u_t. \] (1.13)

Suppose the HP-filter is a good approximation to a low-pass filter, and note that a covariance stationary variable can be decomposed into its frequency components. Then, equation (1.20) can be written as

\[ \Delta p_l^f + \Delta p_h^f = c + \alpha_1 \Delta m_l^f - \alpha_1 \Delta y_l^f + \alpha_1 \varepsilon \Delta i_l^f + \alpha_1 \Delta m_h^f + u_l^1. \] (1.14)
Since $\varepsilon \Delta i_t^f$ is an estimate of the change in low-frequency velocity, the three low-frequency terms are an estimate of low-frequency inflation. Thus, these three terms should be able to explain the low-frequency inflation term well. What is left, then, to be explained is high-frequency fluctuations in prices. The only high-frequency variable on the right-hand-side left to explain short-term inflation is money growth. The following empirical estimations are less restrictive. They allow all variables to explain inflation at all frequencies as all frequency bands are included for each variable.

6 Regression Analysis of Money and Inflation

What matters for policy is the headline inflation rate rather than some frequency component of it. Therefore, this section runs regressions on the unfiltered month-on-month inflation rate. There are three main questions to answer: First, this section aims to find the frequency band that is most important for the relation between money and inflation. In particular, the question is whether money at business cycle frequencies contains information over headline inflation. Second, the regression analysis tries to identify whether there have been changes in the relationship between the two variables over time. Finally, the model allows to test for the two-pillar Phillips curve and controls for changes in velocity.

6.1 Frequency Dependence in the Relationship

The general model to be estimated, given in equation (1.9), does not restrict the frequency components of the different variables to only contemporaneously affect headline inflation. As a first step, however, the coefficients to be estimated, i.e. $\tilde{\beta}_i^f(L)$ are restricted to be contemporaneous only, so the restriction $\tilde{\beta}_i^f(L) = \tilde{\beta}_i^f L^0$ is imposed on equation (1.9). In a second step, regressions are run for lagged money as the graphical analysis suggests that there may be certain time shifts in the correlations.

A bandpass filter extracts the frequency components of a time series by computing a moving average with weights consisting of cosin terms. A bandpass filter has a phase shift of
zero, so the frequency component at time $t$ corresponds to the time series’ observation at time $t$. Hence, equation (1.8) can be considered as contemporaneous. As mentioned above, however, empirical approximations to the optimal filter may exhibit certain phase shifts. Moreover, a one-sided filter allows some interpretation with respect to the direction of the correlation. Therefore, regressions are run separately for two-sided and one-sided filters.

Table 1.3: Regression Results

<table>
<thead>
<tr>
<th>Model</th>
<th>two-sided Filter</th>
<th>one-sided Filter</th>
<th>Money lagged by 2 years</th>
<th>Money lagged by 3 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{m}_t^{(0-1.5y)}$</td>
<td>-0.020</td>
<td>-0.009</td>
<td>-0.009</td>
<td>0.003</td>
</tr>
<tr>
<td>$\tilde{m}_t^{(1.5-3y)}$</td>
<td>-0.138</td>
<td>-0.210</td>
<td>-0.076</td>
<td>-0.134</td>
</tr>
<tr>
<td>$\tilde{m}_t^{(3-5y)}$</td>
<td>-0.123</td>
<td>-0.488</td>
<td>0.499</td>
<td>0.708*</td>
</tr>
<tr>
<td>$\tilde{m}_t^{(5-8y)}$</td>
<td>0.180</td>
<td>0.194</td>
<td>0.903**</td>
<td>0.029</td>
</tr>
<tr>
<td>$\tilde{m}_t^{(8-\infty)}$</td>
<td>0.439***</td>
<td>0.823***</td>
<td>0.887***</td>
<td>0.712***</td>
</tr>
<tr>
<td>$\tilde{y}_t^{(0-1.5y)}$</td>
<td>0.007</td>
<td>0.011*</td>
<td>0.012**</td>
<td>0.013**</td>
</tr>
<tr>
<td>$\tilde{y}_t^{(1.5-3y)}$</td>
<td>-0.030</td>
<td>0.133</td>
<td>0.134*</td>
<td>0.057</td>
</tr>
<tr>
<td>$\tilde{y}_t^{(3-5y)}$</td>
<td>-0.225*</td>
<td>-0.022</td>
<td>0.063</td>
<td>0.048</td>
</tr>
<tr>
<td>$\tilde{y}_t^{(5-8y)}$</td>
<td>-0.577***</td>
<td>-0.524**</td>
<td>-0.334*</td>
<td>-0.166</td>
</tr>
<tr>
<td>$\tilde{y}_t^{(8-\infty)}$</td>
<td>-0.644***</td>
<td>-0.596***</td>
<td>-0.471***</td>
<td>-0.412</td>
</tr>
<tr>
<td>$\tilde{i}_t^{(0-1.5y)}$</td>
<td>0.009</td>
<td>-0.029</td>
<td>-0.025</td>
<td>-0.028</td>
</tr>
<tr>
<td>$\tilde{i}_t^{(1.5-3y)}$</td>
<td>0.037</td>
<td>-0.175</td>
<td>-0.028</td>
<td>-0.062</td>
</tr>
<tr>
<td>$\tilde{i}_t^{(3-5y)}$</td>
<td>0.793*</td>
<td>0.607</td>
<td>0.972**</td>
<td>0.866**</td>
</tr>
<tr>
<td>$\tilde{i}_t^{(5-8y)}$</td>
<td>1.076***</td>
<td>2.137***</td>
<td>1.171**</td>
<td>1.589**</td>
</tr>
<tr>
<td>$\tilde{i}_t^{(8-\infty)}$</td>
<td>1.670***</td>
<td>2.484***</td>
<td>2.342***</td>
<td>2.411***</td>
</tr>
<tr>
<td>$c$</td>
<td>-0.059</td>
<td>0.553**</td>
<td>0.390*</td>
<td>0.467</td>
</tr>
</tbody>
</table>

Adjusted $R^2$ | 0.3128 | 0.2441 | 0.2821 | 0.2544 |
Number of Obs | 576 | 576 | 563 | 551 |
Durbin-Watson | 1.3506 | 1.2536 | 1.3029 | 1.2394 |
QLR statistic | 93*** | 135** | 91*** | 145*** |

These estimations are run for the sample from 1960/01 to 2007/12. Significance levels reported refer to Newey West standard errors (12 lags). ***, **, * denote significance at the 1, 5, and 10% levels. The tilde above each variable indicates that it is a frequency component of the time series. The superscript within the parenthesis indicates the time correspondence of the frequency band.
Contemporaneous Regressions

Each of the three explanatory variables enters the regressions in the form of five frequency components which reflect fluctuations of less than 1.5 years, 1.5 to 3 years, 3 to 5 years, 5 to 8 years and more than 8 years. In line with the quantity theory, we would expect a positive impact of money and a negative impact of output on inflation. On the other hand, the Phillips curve relationship would call for a positive impact of the output gap on inflation. If the interest rate is a good proxy for velocity, then theory would suggest a positive coefficient.

Table 1.3 reports the regression results. The significance levels reported refer to Newey West standard errors allowing for 12 lags. The sample spans from 1960 to the end of 2007. However, as mentioned above, the frequency components are computed using the entire sample of each individual variable. In the first column, the frequency components are computed using the two-sided filter as in the graphical analysis. It suggests that money is important for inflation only in the long run or at frequencies that correspond to more than 8 years. Output has a positive sign only at the highest frequencies corresponding to seasonality patterns, and it is very small. At business cycle frequencies, it is negative but not statistically different from zero. At lower frequencies, the relation postulated by the quantity equation is found. The coefficient estimates on the interest rate confirm Reynard (2006, 2007) who suggested that there needs to be some sort of adjustment for equilibrium velocity. In a regime of lower interest rates, velocity is lower and hence the same money growth rate is associated with lower inflation rates. The fact that the low-frequency components of the interest rate are highly significant is evidence that interest rates at low frequencies can capture shifts in equilibrium velocity quite well. Interestingly, none of the variables is capable of explaining inflation at high frequencies corresponding to less than three years.

The negative relation between inflation and output reminds of the fact that it is only money growth in excess of output growth that is typically thought to cause inflation. Regression results using the frequency components of excess money to explain inflation turn out to be weaker, however. The obvious reason for this is that, as shown below, money performs best when lagged by 2 to 3 years, whereas output should enter the regression contemporaneously.
Two-sided filter regressions may capture feedback effects from the inflation rate to the money growth rate. Therefore, the second column of Table 1.3 reports the estimates produced from one-sided filtering. Money enters significantly only at the lowest frequency, but the size of the coefficient almost doubles. Output now enters significantly positive at the highest frequency which suggests a similar seasonality pattern of output growth and inflation. Moreover, the coefficient at 1.5 to 3 years has reached positive territory but remains very small and insignificantly different from zero. Quantitatively, money becomes more important at low frequencies when a one sided filter is used. This may indicate that money leads inflation.

As Nelson (2003) notes, many studies fail to find the relationship by restricting it to be contemporaneous. Therefore, the next section allows for a lag. The lagged regressions also use a one-sided filter. This choice was taken not only because results are better interpretable from a one-sided filter, but also because the one-sided frequency components were found to even outperform their two-sided counterparts, as the comparison of the two first columns in Table 1.3 suggest. A symmetric two-sided filter does not cause a phase shift of the filtered series. The computation of the frequency components in this study, however, sacrifices the symmetry of the filter in favor of the precision of the estimates. To obtain as accurate estimates as possible, the filter employed uses all information available at all times. Thus, in the first half of the sample the frequency components contain a forward bias as the frequency component is estimated only from subsequent observations. Likewise, the end of the sample exhibits a backward looking bias. In the spirit of precision, the one-sided filter is constructed to incorporate all information available at all times. But, because it is a one-sided filter, the information is restricted to the past. As a consequence, the frequency components contain relatively more information of the past towards the end of the sample. The graphs in Figure 1.6 give evidence that money leads inflation by more (about 3 years) in the most recent subsample than in the early years (about 2 years). Note that this should not be the product of filtering as both variables were filtered the same way so that the phase shifts balance. Because a one-sided filter induces an increasing the phase shift, it may perform better in the regression analysis.
Allowing for a Lag

As mentioned above, the general model given in equation (1.9) is a distributed-lag model of the frequency components. However, serious multicollinearity issues arise with several lags of each frequency component entering the regression. This is true especially for low-frequency components as these are highly autocorrelated. The graphical analysis above suggests that money growth leads inflation by 2 to 3 years. In line with these findings, the regression is run with the frequency components of money entering with a lag of 2 and 3 years. Column 3 and 4 of Table 1.3 report the respective results. With a lag of two years, money becomes highly significant at the lower business-cycle frequencies corresponding to 5 to 8 years. With a lag of three years, the explanatory power of money shifts even to those components that correspond to fluctuations of 3 to 5 year length. Thus, the patterns visible in the graphs can indeed be confirmed by regressions on headline inflation. This is evidence that money matters for inflation not only in the long run, but also at frequencies well within the business cycle.

With money lagged by two years, the output fluctuations of 1.5 to 3 years’ duration impact on inflation. Thus, this is evidence for the Phillips curve relationship. With a lag of three years, the explanatory power of output growth largely disappears. In general, the evidence for a Phillips curve type relation is rather weak. Possibly, the Phillips-curve effect is captured by the interest rate, as a high interest rate often goes hand-in-hand with a positive output gap, which puts upward pressure on prices. Neighboring frequency components of a variable are uncorrelated by definition. Correlation could only arise due to leakage, which is very small as shown in Figure 1.9 in appendix B. However, the frequency components across variables may be correlated within certain frequency bands. Such multicollinearity issues may make it difficult to clearly disentangle the effects of each variable from the others’. Table 1.4 therefore shows the correlations among the variables at different frequencies. Frequency components of the interest rate do exhibit quite large correlations with other variables. It is for example highly negatively correlated with money at frequencies between

\[ \sum_{f=1}^{n} \tilde{\beta}_f(L) \in (1.9) \text{ in an arbitrary way. However, as mentioned in the graphical analysis, the correlation between output and inflation seems to be contemporaneous. Moreover, regressions show a clear deterioration of the explanatory power of output when it is lagged.} \]

11It may not be fair to only lag money. Moreover, lagging money restricts the coefficients \[ \sum_{f=1}^{n} \tilde{\beta}_f(L) \] in (1.9) in an arbitrary way. However, as mentioned in the graphical analysis, the correlation between output and inflation seems to be contemporaneous. Moreover, regressions show a clear deterioration of the explanatory power of output when it is lagged.
Table 1.4: Correlation of Frequency Bands

<table>
<thead>
<tr>
<th></th>
<th>0 to 1.5</th>
<th>1.5 to 3</th>
<th>3 to 5</th>
<th>5 to 8</th>
<th>8 to ∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>m and y</td>
<td>-0.1833</td>
<td>-0.1213</td>
<td>0.1907</td>
<td>0.2105</td>
<td>-0.0248</td>
</tr>
<tr>
<td>m and i</td>
<td>-0.0094</td>
<td>-0.2215</td>
<td>-0.2444</td>
<td>-0.6044</td>
<td>0.1509</td>
</tr>
<tr>
<td>y and i</td>
<td>0.0166</td>
<td>0.6240</td>
<td>0.7381</td>
<td>0.3615</td>
<td>0.2985</td>
</tr>
</tbody>
</table>

5 and 8 years. Moreover, its correlation with output growth is very large at business cycle frequencies. This may explain why it is difficult to find the positive effect of output at those frequencies.\textsuperscript{12}

To conclude, the regression results suggest that money is most important for inflation in the long run, and there is only limited explanatory power in fluctuations corresponding to the business cycle. However, if the comovement between money and inflation, e.g. due to changes in velocity, shift between frequency bands over time, or if the lag structure evolves, then the regressions above are not able to capture such changes.

6.2 Time-Variance in the Relationship

The graphs in Section 4 indicate that the relationship between money and inflation changed with the beginning of the Volcker era around 1980. To test this, breakpoint tests are run on the regressions above. As a second step, a dummy variable is included to allow for a different coefficient during that time.


Breakpoint Tests

Chow’s (1960) breakpoint test provides a means to test whether a coefficient has changed at a given point in time. The Chow breakpoint test is based on a comparison of the sum of

\textsuperscript{12}When the interest rate enters the regression as an unfiltered measure to avoid such multicollinearity, the results for output remain almost unchanged while the goodness of fit of the regression deteriorates considerably.
squared residuals obtained by fitting a single equation to the entire sample with the sum of squared residuals obtained when separate equations are fit to each subsample of the data. The F-statistic

\[ \xi_W = \frac{(\mathbf{u}'\mathbf{u} - u'_1u_1 - u'_2u_2)/k}{u'_1u_1 + u'_2u_2)/(T - 2k)} \] (1.15)

is based on the comparison of the restricted and the unrestricted sum of squared residuals, where \( \mathbf{u}'\mathbf{u} \) is the restricted sum of squared residuals and \( u'_iu_i \) is the sum of squared residuals of each subsample. \( k \) is the total number of parameters in the equation. The F-statistic can be extended to more than one breakpoint. However, the date of the breakpoint must be known in order to apply the Chow test. With unknown breakpoint, the Chow statistic can be run for a large number of possible breakpoints. With \( \tau \) denoting the fraction of the sample, the Quandt likelihood ratio (QLR) statistic

\[ \xi_Q = \max_{\tau_1 \leq \tau \leq \tau_2} \xi_W(\tau) \] (1.16)

gives the largest value of the Chow statistics running from \( \tau_1 \) to \( \tau_2 \). The breakpoint lies somewhere between the end of the 1970s and the mid 1990s. This information should be incorporated to maximize power of the test. In order not to be too restrictive, only the dates between 1975 and 2000 are included in the test. The respective fractions of the sample are \( \tau_1 = 0.31 \) and \( \tau_2 = 0.78 \). \( \xi_Q \) no longer follows a \( \chi^2 \) statistic. The critical values are originally tabulated in Andrews and Ploberger (1994) and revised in Andrews (2003). The 1%, 5%, and 10%-critical values for the QLR statistic, with \( k = 16 \), are 40.87, 35.15, and 32.36, respectively. The two rows at the bottom of Table 1.3 indicate the results of such a breakpoint test. The statistics clearly exceed the respective critical values, indicating that there was a structural break somewhere at the beginning of the 1980s.

**Inclusion of Dummy Variables**

The graphical analysis above suggests that there is a close relation between money and inflation at business cycle frequencies, but that this relation broke down during the Volcker era. Whether it appeared again during the past 20 years is not clear from the graphs. This would mean that there was a temporary structural break during approximately one and a
half decades. In order to shed light on this, dummy variables are added to the regressions above.

In a first step, the different frequency components of money are interacted with a dummy variable for the time interval between 1980 and 1995 and added to the regression. Table 1.5 gives the results in the first column. The coefficient on the frequency band corresponding to 3 to 5 years increases and becomes significant. Moreover, the coefficient on the lower business cycle frequencies increases further. Hence, money affected inflation differently during and after the disinflation period. And, controlling for this period suggests a very strong comovement at business cycle frequencies. There is only very weak evidence that the relationship had moved to the higher frequencies, however. The respective coefficient turns from negative to positive, but it remains statistically insignificantly different from zero.

In a second step, another dummy is interacted with money to allow the relationship to be different since 1995. The results are depicted in the second column of Table 1.5. The 3 to 5 year frequency component of money remains significant. But when interacted with the dummies for both later subsamples, its size is strongly reduced. While the relationship is even reversed during the Volcker era and the subsequent years, it is approximately halved since 1995. It seems that part of this long-run correlation has moved to a higher frequency band. In the recent one and a half decades, the most important frequency band seems to be the one corresponding to 5 to 8 years. Moreover, there is the indication that the long term relation between money and inflation has been significantly smaller since 1995. This is in line with Nelson (2003) who argues that “a protracted fall in nominal interest rates, associated with the aftermath of a disinflation, reduces the cost of holding real balances and so generates data characterized by higher money growth, slower velocity growth and unchanged inflation”.

Finally, the last column of Table 1.5 imposes the assumption that the relationship changed with the Volcker era and remained stable since. This regression confirms that the comove-
The first part of the sample. It seems to persist only at the lower business cycle frequencies and $D$ refers to the time intervals 1980 to 1994 and 1995 to 2007, respectively. Significance levels reported refer to Newey West standard errors (12 lags). ***, **, * denote significance at the 1, 5, and 10% levels.

Table 1.5: Dummy Regressions

<table>
<thead>
<tr>
<th>Dependent Variable: $\pi_t$</th>
<th>$m_{1t}^{(0-1.5y)}$</th>
<th>$m_{1t}^{(1.5-3y)}$</th>
<th>$m_{1t}^{(3-5y)}$</th>
<th>$m_{1t}^{(5-8y)}$</th>
<th>$m_{1t}^{(8-\infty)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^1 \cdot m_{1t}^{(0-1.5y)}$</td>
<td>-0.003</td>
<td>0.034</td>
<td>0.032</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D^1 \cdot m_{1t}^{(1.5-3y)}$</td>
<td>-0.129</td>
<td>-0.241</td>
<td>-0.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D^1 \cdot m_{1t}^{(3-5y)}$</td>
<td>0.676*</td>
<td>0.743*</td>
<td>0.794**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D^1 \cdot m_{1t}^{(5-8y)}$</td>
<td>0.958*</td>
<td>0.636</td>
<td>0.804</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D^1 \cdot m_{1t}^{(8-\infty)}$</td>
<td>0.894***</td>
<td>0.779***</td>
<td>0.892***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D^2 \cdot m_{1t}^{(0-1.5y)}$</td>
<td>-0.020</td>
<td>-0.054</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D^2 \cdot m_{1t}^{(1.5-3y)}$</td>
<td>0.205</td>
<td>0.323</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D^2 \cdot m_{1t}^{(3-5y)}$</td>
<td>-0.928</td>
<td>-0.858</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D^2 \cdot m_{1t}^{(5-8y)}$</td>
<td>-0.263</td>
<td>0.199</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D^2 \cdot m_{1t}^{(8-\infty)}$</td>
<td>0.062</td>
<td>-0.024</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(D^1 + D^2) \cdot m_{1t}^{(0-1.5y)}$</td>
<td>-</td>
<td>-</td>
<td>-0.065</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(D^1 + D^2) \cdot m_{1t}^{(1.5-3y)}$</td>
<td>-</td>
<td>-</td>
<td>0.218</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(D^1 + D^2) \cdot m_{1t}^{(3-5y)}$</td>
<td>-</td>
<td>-</td>
<td>-1.026*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(D^1 + D^2) \cdot m_{1t}^{(5-8y)}$</td>
<td>-</td>
<td>-</td>
<td>-0.014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(D^1 + D^2) \cdot m_{1t}^{(8-\infty)}$</td>
<td>-</td>
<td>-</td>
<td>-0.084</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_t^{(1.5-3y)}$</td>
<td>0.011*</td>
<td>0.013**</td>
<td>0.013**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_t^{(3-5y)}$</td>
<td>0.131</td>
<td>0.144</td>
<td>0.146*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_t^{(5-8y)}$</td>
<td>0.048</td>
<td>0.026</td>
<td>0.018</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_t^{(8-\infty)}$</td>
<td>-0.318*</td>
<td>-0.404**</td>
<td>-0.409**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_t^{(0-1.5y)}$</td>
<td>-0.418**</td>
<td>-0.609***</td>
<td>-0.575***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_t^{(1.5-3y)}$</td>
<td>-0.024</td>
<td>-0.030</td>
<td>-0.030</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_t^{(3-5y)}$</td>
<td>0.027</td>
<td>-0.049</td>
<td>-0.056</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_t^{(5-8y)}$</td>
<td>0.906**</td>
<td>1.034**</td>
<td>1.900**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_t^{(8-\infty)}$</td>
<td>0.937**</td>
<td>1.165***</td>
<td>0.961**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_t^{(0-1.5y)}$</td>
<td>2.247***</td>
<td>2.363***</td>
<td>2.208***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_t^{(1.5-3y)}$</td>
<td>2.317</td>
<td>0.614***</td>
<td>0.491**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.2844</td>
<td>0.3101</td>
<td>0.2959</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Obs</td>
<td>563</td>
<td>563</td>
<td>563</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson</td>
<td>1.3185</td>
<td>1.3885</td>
<td>1.3470</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These estimations are run for the sample from 1960/01 to 2007/12. Money is lagged by 2 years. The dummy variables $D^1$ and $D^2$ refer to the time intervals 1980 to 1994 and 1995 to 2007, respectively. Significance levels reported refer to Newey West standard errors (12 lags). ***, **, * denote significance at the 1, 5, and 10% levels.

The movement between money and inflation at frequencies corresponding to 3 to 5 years is limited to the first part of the sample. It seems to persist only at the lower business cycle frequencies.
and in the long run.

To sum up, this method is partially able to identify the change in the comovements over time as suggested by the graphical analysis in Section 4. However, the relationship at frequencies corresponding to 3 to 5 years is strongest in the early subsample ending 1979, it disappeared during the Volcker era and might have come back at a weaker level. At the lowest frequency, a very similar pattern reveals: the relation between money and inflation is smaller in the latest subsample starting 1995. The results of the empirical analysis shows clearly that the relationship between money and inflation is not stable over time. It moves both with respect to frequencies as well as to the lag structure. A linear model such as a vector autoregression imposes a constant lag structure over time. The estimates will therefore be biased towards zero if the assumption of time stability is violated. In addition, the analysis shows that to identify the comovement between money and inflation, different frequency bands should be taken into account.

7 Conclusion

Money and inflation are most often shown to correlate only in the “long run”. But there is very little precision as to the actual timing of the comovement. From a policy perspective, however, it is important to have a better understanding of the timing of such a comovement. Therefore, this paper studies the correlation between money and inflation at different frequency bands. Money and inflation are found to correlate strongest at low frequencies, i.e. at cycles of more than 8 years’ length. In the 1980s and early 1990s, when the correlation between money and prices is typically found to break down, graphical analysis suggests that the correlation might have shifted into higher frequency bands. Yet, the regression analysis is unable to replicate this result. But when controlling for that period, the regression results show a strong comovement between money and inflation at frequencies that correspond to the business cycle, i.e. of 3 to 8 years’ fluctuations. Moreover, the results suggest that, after a break during the Volcker era and the subsequent years, the correlation returned in the mid 1990s. The more recent relationship is slightly weaker and more pronounced at lower frequencies, however. Since the mid 1990s, however, the corre-
lation concentrates at higher frequencies that correspond to 5 to 8 years. These results not only support the evidence that there is indeed a comovement between monetary aggregates and inflation, but that this comovement changes over time with respect to both the lead-lag structure and with respect to the relevant frequencies. While linear models may suffer from biased estimates due to the absence of time stability, spectral techniques as applied in this study seem to be a suitable instrument to further investigate this relationship.
A Some Properties of Bandpass Filters

A bandpass filter is a lag operator that attributes certain weights to leads and lags of a variable. The low-pass filter passes all movements in a variable that are attributable to frequencies below a chosen threshold, $\omega^a$. It can be shown that the optimal low-pass filter

$$a(L) = \sum_{j=-\infty}^{\infty} a_j L^j$$

(1.17)

where $L$ denotes the lag operator, has weights that take the form

$$a_j = (2\pi)^{-1} \frac{1}{ij} e^{i\omega_j j} |_{\omega^a} = \begin{cases} \frac{1}{j}\sin(\omega^a j) & \text{for } j \neq 0 \\ \frac{\omega^a}{\pi} & \text{for } j = 0 \end{cases}.$$  (1.18)

Equivalently, the filter $(1 - a(L))$ is a high-pass filter passing all frequencies larger than $\omega^a$. The low-frequency and high-frequency components $a(L)x_t$ and $(1 - a(L))x_t$ of some variable $x_t$ are uncorrelated by construction. Moreover, filters are additive in their gains, so the low-frequency and high-frequency components $a(L)x_t$ and $(1 - a(L))x_t$ add up to the original time series $x_t$. Suppose $a(L)$ is a low-pass filter that passes on frequencies lower or equal to $\omega^a$. Let $b(L)$ be another low-pass filter that passes on frequencies up to $\omega^b$, where $\omega^a < \omega^b$. Then, the bandpass filter $c(L) = a(L) - b(L)$ passes all frequencies $\omega^c$ that lie between the lower and the upper threshold, so $\omega^a < \omega^c < \omega^b$. Thus, using bandpass filters, we can decompose a time series into its different frequency components.

B Bandpass Filter Gain

There is always the issue of end-of-sample accuracy of filters. The bandpass filter applied in this study uses all the information available for each data point. So as an approximation to the optimal filter in (1.17), the filter weights are truncated at the start and at the end of the sample. Thus, at the beginning of the sample, the filter is mainly forward-looking. Towards the end of the sample, the filter is backward-looking. Likewise, the gain of the filter varies among observations $t$. Figure 1.8 shows the gain of the bandpass filter. In the left panel, the gain is plotted for the filter at each observation $t$. In the right panel,
the gains of the first and last 5% of observations are dropped. This shows that the filter provides a reasonable approximation to the ideal filter after dropping approximately 5% of observations at both ends.

**Figure 1.8:** The Gain of the BP-Filter at Each Observation

The entire sample of each variable is used to compute the frequency components. As regression starts in 1960, and inflation, industrial production, and interest rates are available way back, it is only the frequency components of money that might suffer from poor end-of-sample properties of the filter. The data is collected through June 2011, regressions are run only up to the end of 2007. So at the end of the sample, 42 observations are dropped. Despite the fact that there are most likely enough data dropped at both ends of the sample, the impact of dropping data can easily be analyzed empirically. I computed different frequency components for inflation, applying the bandpass filter to 11 different samples: the full sample, cut 1 year at each end, cut 2 years at each end, etc. The following results obtain: At high frequencies corresponding to less than 4 years, as the filtered series are almost identical. Hence, there is no reason to cut any data. The deviations are still minor for frequency components corresponding to 4 to 8 years. Even for those corresponding to 8 to 15 years, the differences are relatively small. However, for the very low frequencies, there are shifts in the trend that are quite large. This is true even when the data is detrended prior to filtering. And it is also true whether detrending is undertaken in the time or in the spectral domain, as Corbae et al. (1997) suggest. To conclude, end-of-sample properties are certainly not an issue for frequencies corresponding to cycles of less than 15 years.
For the very low frequency components, uncertainty about the precision of the frequency component is larger. This is true not only with respect to poor end-of-sample properties of the filter, but also with respect to prior manipulation of the data with respect to trends. I do the best I can by letting the filter use all the information available at each point in time.

Another empirical issue related to the gain is leakage. Since the filter in an empirical application is always an approximation to the ideal filter, it is imprecise. The less observations, the larger is the leakage, which refers to the near-by frequencies outside of the particular passband that are partially passed by the filter due to its imprecision. Figure 1.9 shows the gain of the bandpass filter for the frequency bands relevant for the empirical section of this paper. The vertical dotted lines indicate the exact frequencies $\omega$ of the thresholds between the frequency bands. For example, $\omega = 0.105$ denotes the period $2 \cdot \pi/\omega = 60$. Since we have monthly data, this corresponds to 5 years. So when the gain of a certain passband crosses the corresponding frequency, it means that bits outside of the passband are passed by the filter. To compute these gains, a sample size of 564 observations is used, which corresponds to monthly data of 47 years, i.e. from 1960 to 2007. The Figure clearly shows that there is some leakage when the bandpass filters are constructed without dropping intermediate frequencies. However, a problem for the present analysis arises only if leakage leads to multicollinearity, such that the contributions of the separate frequency bands can no longer be clearly disentangled. Looking at the correlation matrix of the frequency bands confirms for each variable that the correlations are rather small. Hence, leakage can be ignored.

C End-of-Sample Properties of Bandpass Filter

In theory, the ideal bandpass filter is a symmetric weighted average of an infinite number of leads and lags. In practical applications, there are essentially two ways to approximate the ideal filter: First, a symmetric filter with relatively few leads and lags can be computed. Second, an asymmetric filter can be constructed that uses all data points available at each point in time. The former method requires to drop data at both the beginning and at the end of the sample. The latter method potentially induces a phase shift. This paper uses the second method. The entire sample of each variable is used to compute the frequency
components. Hence, even if the final regression is run starting only in 1995, the frequency components of M2 use data back to 1959. However, the filter might still suffer from poor end-of-sample properties, especially in those samples starting in 1960. Hence, it may be necessary to cut some observations at least at the end of the sample. In order to find out how much needs to be cut, different frequency components for inflation were computed, applying the bandpass filter to 11 different samples: the full sample, 1 year cut at each end, 2 years cut at each end, etc. The following results obtain: At high frequencies (0-4 years), there is no reason to cut data, as the filtered series are almost identical. The deviations are still minor for frequency components of 4-8 years, and even for those corresponding to 8-15 years, the differences are relatively small. For the very low frequencies, there are shifts in the trend that are quite large. This is true even when the data is detrended prior to bandpass filtering (this is true for detrending both in the time and in the spectral domain).

D  Velocity Adjustment of Money

In a first step, Reynard (2006, 2007) estimates interest rate elasticity of money demand from

\[ v_t \equiv m_t - p_t - y_t = c + \varepsilon \cdot \ln i_t^{10y}. \]  

(1.19)
Then, he runs regressions of the form

\[ \Delta p_t = c + \alpha_1 \tilde{\mu}_t + \alpha_2 \tilde{\mu}_{t-4} + \alpha_3 \tilde{\mu}_{t-8} + u_t \quad (1.20) \]

where

\[ \tilde{\mu}_t = \Delta m_t - \Delta y_t^* + \varepsilon \Delta i_t^* \quad (1.21) \]

with \( * \) denoting a HP-filtered variable, so that \( \Delta y_t^* \) is the potential growth rate, or the long-run growth rate, and \( i_t^* \) denotes some sort of equilibrium interest rate. So money growth in excess of potential output growth is adjusted by equilibrium velocity. The goal is to see whether money can explain inflation in the short term after imposing long-term restrictions.

The regression equation (1.20), skipping the lagged regressors, can be thus written as

\[ \Delta p_t = c + \alpha_1 \Delta m_t - \alpha_1 \Delta y_t^* + \alpha_1 \varepsilon \Delta i_t^* + u_t^1 \quad (1.22) \]

Suppose the HP-filter is a good approximation to a low-pass filter, and note that a covariance stationary variable can be decomposed into its frequency components. Then, equation (1.20) can be written as

\[ \Delta p_t^{lf} + \Delta p_t^{hf} = c + \alpha_1 \Delta m_t^{lf} - \alpha_1 \Delta y_t^{lf} + \alpha_1 \varepsilon \Delta i_t^{lf} + \alpha_1 \Delta m_t^{hf} + u_t^1. \quad (1.23) \]

Since \( \varepsilon \Delta i_t^{lf} \) is an estimate of the change in low-frequency velocity, the three low-frequency terms are an estimate of low-frequency inflation. Thus, these three terms should be able to explain the low-frequency inflation term well. What is left, then, to be explained is high-frequency fluctuations in prices. The only high-frequency variable on the right-hand-side, however, is money growth. Surely, high-frequency fluctuations in money growth are not what we would expect to explain short-run fluctuations in inflation.
Chapter 2

Liquidity Effects of Quantitative Easing on Long-Term Interest Rates

Abstract

This paper argues that the expansion in reserves following recent quantitative easing programs of the Federal Reserve may have affected long-term interest rates through liquidity effects. The data lends some support for liquidity effects, in that reserves were negatively correlated with long-term yields at the zero lower bound. Estimates suggest that between January 2009 and 2011, 10-year US Treasury yields fell 46-85 basis points as a result of liquidity effects. The liquidity effect is separate from the portfolio balance effect of the change in the public supply of Treasury bonds, which is estimated to have reduced yields by another 20 basis points during that period.
1 Introduction

An increase in the money supply is usually expected to reduce short-term interest rates. This phenomenon, known as the liquidity effect, finds empirical support in the literature. It is commonly held that when short-term interest rates reach the zero lower bound (ZLB), the liquidity effect disappears. This is because the short-term liquid assets typically bought in open market operations (OMO) form a perfect substitute for money when short-term interest rates are at zero. However, banks also demand and hold long-term assets with strictly positive yields. OMOs in long-term bonds could hence still have a liquidity effect. The increase in zero-yielding reserves associated with the purchases of longer term bonds in OMOs at the ZLB, induces banks to search for a higher return on their assets. They hence raise their demand for positive yielding medium to longer-term assets, such as government bonds. The higher demand for these assets, in turn, reduces their yield. In this way, an increase in reserves should be related to lower long-term yields at the ZLB.

That the large scale asset purchases conducted by the Federal Reserve at the ZLB may have been transmitted to yields through such liquidity effects has been overlooked. Chart 2.1 shows how non-borrowed reserves held with the Federal Reserve, and long-term Treasury yields, moved in the same overall direction during the recent ZLB period. There could be many reasons for these variables to have been correlated during that period. One candidate is that the relationship is partly causal.

Instead of focusing on the effect of reserves, the currently very active literature has focussed on the effect of the assets purchased in these operations. Thus, the expansion in reserves during the past years came about partly through outright purchases of Treasury bonds. When the central bank buys a government bond, it augments the asset side of its balance sheet with this bond, and augments the liability side with the corresponding amount of reserves. The liquidity effect is the impact of an expansion of the central bank’s liabilities on bond yields, irrespective of the type of asset the central bank buys. The previous literature argues that the change on the asset side of the balance sheet, i.e. the outright purchase of a specific asset, such as a specific maturity government bond, constitutes the main channel through which quantitative easing influences the yield on this bond, as the
quantity of the purchased bond in the market changes.

In this paper, we lay out the argument for why liquidity effects may have been at work, and take a first look at the data to offer some initial empirical support for the hypothesis that liquidity effects constitute an additional and potentially important transmission channel, increasing the overall impact of the asset purchases on long-term rates. Specifically, we add reserves to a standard regression specification for the 10-year Treasury yield used for testing for supply effects. We allow the effect of reserves on yields to differ between the present ZLB period and the pre-ZLB period in this regression.

We find that (i) the realized yield on the 10-year Treasury bond has been lower during the recent ZLB period than what the standard regression used for testing for supply effects predicts and (ii) adding non-borrowed reserves in percent of GDP to the regression results in a significantly negative effect of reserves on yields at the ZLB. The estimated correlation between reserves and yields suggest that yields were 46-85 basis points lower during the ZLB period due to liquidity effects of the monetary expansion. This liquidity effect is in addition to the supply effect, meaning that the overall impact of the asset purchases at the ZLB was larger.

The next section offers some theoretical considerations regarding how central bank asset purchases might affect the yield of bonds through liquidity effects as well as supply effects. The empirical investigation of liquidity effects is set up in the subsequent section. The final section concludes.

2 How Do Central Bank Asset Purchases Affect Interest Rates?

How should we expect a change in commercial banks’ reserves held at the Fed to affect long-term interest rates, and how important has the effect been during the ZLB period of quantitative easing (QE) programs? To see this, consider three assets: reserves $M$ with
zero yield, a short-term treasury bill $A$ with yield $r_A$, and a long-term treasury bond $B$ with yield $r_B$.

The recent literature on the effects of QE on long-term yields have focused on the impact of changes in the quantity $B$, i.e. Fed’s purchases of long-term bonds and government supply of Treasuries, on $r_B$, which is referred to as portfolio balance or supply effect, as well as on the signaling effect which is not discussed here (see next Section). This approach is based on portfolio balance models incorporating different assets with imperfect substitutability through financial frictions. In these models, a change in the relative supply of $A$ and $B$ affects their relative yield. Thus, in a portfolio balance model with market imperfections, such as segmented markets and assumptions of preferred habitat, a central bank purchase of an asset must increase the price of the asset in question, in order to make market participants accept holding less of the respective asset.\(^1\) Hence, the yield of the asset falls. Recent empirical research, such as Kuttner (2006), Gagnon et al. (2010), Greenwood and Vayanos (2010), Hamilton and Wu (2010b), Neely (2010), and D’Amico and King (2011), shows that central bank purchases of specific assets influence their yields through so-called supply effects.

This so-called supply effect relates to the asset side of the central bank’s balance sheet. The models do not incorporate the equivalent increase in the liabilities side of the balance sheet, namely reserves. The effect of changes in reserves, or liquidity effect, has been addressed in frameworks that have modeled the private sector’s choice between reserves, $M$, and a short-term bond, $A$. In practice, the liquidity effect arises from the fact that the portfolios of banks involved in OMOs include more reserves than before the sale of assets to the central bank. A higher level of reserves is empirically found to be related to a fall in short-term interest rates, in normal times. Specifically, the liquidity effect has been found to reduce short-term interbank deposit rates such as the Federal funds rate in the US, and short-term government bond yields; see for example Cochrane (1989), Gordon and Leeper (1994), Christiano and Eichenbaum (1995), Christiano and Eichenbaum (1992a), Hamilton (1997), Bernanke and Mihov (1998), Carpenter and Demiralp (2008), Thornton (2008a),

\(^1\)Recent examples of preferred-habitat models include Hamilton and Wu (2010b) or Greenwood and Vayanos (2010).
Liquidity effects have been modeled using segmented market assumptions. Examples of such models are Grossman and Weiss (1983), Christiano and Eichenbaum (1992b), Lucas (1990), or Dotsey and Ireland (1995). In line with a cash-in-advance constraint, a central bank’s lump sum monetary injection cannot immediately be used as transaction balances by the private agents receiving the injection. It therefore pushes up the price of the alternative asset available to market participants.

In practice, the mechanisms in models of the liquidity effect are similar to the ones at work in the portfolio balance approach. In fact, liquidity effects can be thought of as supply effects of reserves. The only differences are the purpose of asset holdings, and the types of frictions. Banks, holding more reserves after an OMO, seek to trade some of these additional reserves for positive yielding assets. Given that the yield on reserves M is fixed at zero, the higher demand for A and B should put upward pressure on the price of both of these assets and thus reduce their yields, in contrast to the supply effect which only pertains to the yields of the specific asset bought in the OMO or to its close substitutes. Indeed, Cochrane (1989) finds that liquidity effects influence the entire yield curve. An important difference between supply and liquidity effects is that while the former suggest a long term link between the supply of an asset and its yield, liquidity effects suggest a link between reserves and interest rates which is temporary. The reason is that an increase in reserves will only affect yields as long as it is in excess of required reserves. But with time, higher excess reserves transform into increased bank credits, which in turn reduce excess reserves (turn excess reserves into required reserves). This is in line with the empirical literature. For example, Christiano and Eichenbaum (1992a) and Christiano and Eichenbaum (1995) find temporary liquidity effects in short rates lasting from one to 20 quarters.

Both the supply effect and the liquidity effect rely on imperfect asset substitutability and should be addressed jointly in the same framework. This would allow for an evaluation of the total effect of QE on long-term yields. Tobin (1958) has suggested a portfolio analysis with money and bonds of different maturities. Andres et al. (2004) provided a theoretical framework including these assets. To understand how the different effects of an OMO can be combined, consider again the three assets $M, A$ and $B$. A supply effect occurs when the
supply of $B$ changes, for example when the government pays back some long-term debt. In this case, the yield $r_B$ decreases. A liquidity effect occurs as a result of a change in the supply of reserves $M$, for example through a "helicopter drop" of reserves on banks. In this case, the "yield" of reserves compared to the yield of other assets has to change. With zero or a low and fixed interest on reserves, this will put downward pressure on the yields of both assets $A$ and $B$.

In normal times, during an OMO, the central bank (CB) buys $A$ with $M$, thus the supply and the liquidity effect both put downward pressure on $r_A$, and the liquidity effect puts downward pressure on $r_B$. Note that we isolate here the supply and liquidity effect from the Fisher effect which can drive yields in the other direction. At the ZLB, when the CB buys $B$ with $M$, both the supply and liquidity effect put downward pressure on $r_B$. The yield on the short-term bill, $r_A$, can be pushed down by the liquidity effect, if it is not already at the ZLB. It is often argued that there cannot be liquidity effects at the ZLB, when $A$ has a zero yield like $M$. But the total supply of zero-yielding assets $M$ and $A$ increases relative to $B$ when the CB buys $B$ in OMO, putting downward pressure on $r_B$ in addition to the supply effect. Finally, note that in the case of the current maturity extension program "Operation Twist II", the CB buys $B$ and sells $A$. In that case there is no liquidity effect which would put downward pressure on both $r_A$ and $r_B$, as the amount of reserves remain constant. Supply effects put downward pressure on $r_B$ and upward pressure on $r_A$.

3 An empirical assessment

As argued above, a central bank asset purchase directly from the market is transmitted through at least three different channels, namely the supply effect, the liquidity effect and through signalling. Signalling effects occur because a central bank asset purchase may signal something about the central bank’s intentions for future monetary policy, and hence, the future short term interest rate path. We hence need a research design which allow us to distinguish these three effects. The following sections present how we approach these issues conceptually and econometrically.


3.1 Identifying liquidity effects

If market participants are informed about asset purchases by the central bank in advance - as has been the case for all three recent large scale asset purchase programs (LSAP) - and if market participants have perfect and complete information, and hence know the impact of these purchases on yields through supply and liquidity effects, then this impact should be discounted immediately at announcement. In contrast, there should be no effect on yields when the transactions actually take place under these assumptions. A number of studies estimating the supply effects of the Fed’s asset purchases have hence looked for announcement effects on yields, using event studies techniques. In order to identify portfolio balance effects from signalling effects at announcement, such studies divide the long-term Treasury yield into a term premium and the expected future short rates using econometric term structure models. Supply effects affect the term premium whereas signalling effects should only affect expected future short rates (see for example Bauer and Rudebusch (2011-21) and the first part of the analysis in Gagnon et al. (2010). Neely (2010) is an exception).

The event study approach does not allow for an identification of liquidity effects. This is because both liquidity and supply effects affect the term premium at the ZLB. Their individual effects hence cannot be identified by simply investigating the reaction of the term premium. There are other problems with the use of event studies in this context. One is that the breakdown between the expected future short rates and the term premium turns out to be very uncertain, and dependent on the type of term structure model estimated, see for example Bauer and Rudebusch (2011-21). Another problem is that the assumption of complete and perfect information of market participants, which is necessary for interpreting the effect of an announcement on yields as containing information about supply and liquidity effects, might be too strong.

Instead, we take a different approach to identification and note that if market participants do not have perfect and complete information, then there is room for some of the liquidity and supply effects of planned transactions to materialize when the actual transactions take place. Signalling effects, on the other hand, should only be present at announcement times. In the time series dimension, what matters for the supply effect is the total supply of the
asset in question, i.e. the total supply of Treasury bonds available to the public. This supply has been affected by the Fed’s purchases during the LSAP, but it has also - and to an even higher degree - been affected by the Treasury’s net issuance of new bonds. For this reason, the public supply of Treasuries has had a source of independent variation, and its correlation with reserves is actually quite small. If there is an effect of reserves on long-term treasury yields when controlling for the public supply of Treasuries, then it should be possible to interpret this effect as a liquidity effect, as separate from a supply effect.

3.2 Methodology and Data

We take as a starting point the time series approach employed by Gagnon, Raskin, Remache and Sack (2010) to assess supply effects in a regression of long-term treasury yields. We add reserves to the regression in order to see if these help explain variation in long-term yields at the ZLB. The effect of reserves is allowed to differ between the pre-ZLB and the ZLB periods. The sample ranges from February 1990 to January 2011. The regressions are carried out in weekly frequency, yielding 112 observations for the ZLB period and 1096 observations for the entire sample. The few explanatory variables for which data is in lower frequency are linearly interpolated (see appendix for details). The baseline regression equation takes the following form:

\[ i_{t}^{(10y)} = \alpha + \beta_1 \cdot D_{t}^{pre-ZLB} R_{t-1} + \beta_2 \cdot D_{t}^{ZLB} R_{t-1} + \beta_3 \cdot D_{t}^{ZLB} + \delta X_t + u_t. \]  

(2.1)

\( i_{t}^{(10y)} \) denotes the average daily yield on 10-year US Treasury bonds over the week following \( t - 1 \). \( R_{t-1} \) denotes the level of non-borrowed reserves held with the Fed at date \( t - 1 \), in percent of GDP. Reserves are thus effectively lagged one week. \( D_{t}^{pre-ZLB} \) and \( D_{t}^{ZLB} \) are dummy variables for the pre-ZLB and the ZLB period, respectively. \( X_t \) contains the control variables described below. We define the ZLB to start in mid December 2008, when the Federal funds target rate was lowered to 0.25%. The interaction of reserves with these

\^[2] All control variables for which data is available on a daily basis enter in averages over the week following \( t - 1 \).

\^[3] We use the percentage of GDP to make reserves comparable in specification to the specification of the Treasury supply, see below. It should be mentioned that the results obtained in this paper are not driven by changes in GDP. When carrying out the regressions using nominal reserves, all results remain.
dummies allows the liquidity effect to differ between normal times and at the ZLB. The level of the ZLB-dummy is included to capture a change in the interest rate level during the ZLB period due to factors that cannot be fully accounted for by the explanatory variables.

The supply of Treasury securities to the public in percent of nominal GDP is included to capture supply effects. Treasury supply is measured as the total supply minus the Fed’s holdings of Treasury securities, as reported in the System Open Market Account (SOMA). We include all maturities above one year in order to account for the transmission of changes in the supply of Treasury bonds to other maturities through substitution effects.\(^4\) Since changes in the maturity structure of the total supply could matter for the supply effect on longer yields, we also control for the average maturity of Treasury bonds.\(^5\)

The Federal funds target rate is included to account for the current interest rate level. Both reserves and the long-term yields may have been driven by weakening economic conditions and resulting changes in expected future monetary policy during the ZLB period. To account for expected future short term interest rates, and hence, expected future monetary policy path, we include the expected change in the 1-year rate one year ahead (see appendix for details). The unemployment gap and the inflation rate of core CPI are additionally included to capture the effect of current economic conditions and the business cycle. Backus and Wright (2007) show that the term premium is counter-cyclical because risk appetite tends to be larger in booms than during busts. We hence expect the unemployment gap to be positively related to the yield, since we control for changes in the expected future path of short-term rates. The 6-month realized volatility of the Treasury yield itself is included to take account of uncertainty about expectations which may boost the demand for safe assets.\(^6\) Uncertainty about inflation expectations may affect longer-term interest rates

---

\(^4\)See for example D’Amico and King (2011) on the substitution effects between different maturity Treasury bonds.

\(^5\)Hamilton and Wu (2010b) provide data on total and public supply of Treasury securities for each maturity. Gagnon et al. (2010) additionally subtract holdings of foreign official agencies, which is not done in the present analysis.

\(^6\)The choice of the volatility measure is inconsequential for our results. We run the same regressions using the logarithm of the VIX instead of the realized volatility, with no change to the findings.
through its effect on the term premium. To control for this uncertainty, we include the interquartile range of long-term inflation expectations obtained from the Michigan Survey of Consumers.

In additional robustness tests, inflation expectations from the Cleveland Fed are used to run the regression on the real long-term yield. The regression is also run for the term spread between 10-year and the 3-month interest rate, as well as for the nominal and real 5-year rates. Moreover, we control for announcement effects by including two dummies. The first is an event dummy that captures the Fed’s announcements related to future monetary policy, i.e. the LSAP programs (QE1 and QE2). The particular events are listed in Table 2.7 in the appendix. The choice of the specific events which would increase the market’s expectation of future asset purchases is in line with recent case studies, e.g. Gagnon et al. (2010) and Neely (2010). The second dummy is a Jackson Hole dummy, taking the value one from the date of the famous speech (27th August 2009) and until the end of the sample. There is a widespread belief that with Bernanke’s speech at Jackson Hole, the entire effect of QE2 was immediately priced in, so that the actual purchases following the announcement had no effect on yields anymore.

3.3 Results

Table 2.1 presents the results of the pre-crisis regression mimicking the sample used in the previous literature on supply effects. The first column reports the results excluding both the supply of Treasury securities to the public and reserves. The second column reports the results when the Treasury supply is added. The results are similar to those in the recent literature. Specifically, the size of the supply effect is very close to the estimate in Gagnon et al. (2010).\footnote{The coefficients on the other explanatory variables deviate somewhat from those found in Gagnon et al. (2010). The main reason is that our sample is slightly shorter because of the lack of access to Treasury supply data before 1990.}

How well does this specification perform for the ZLB period? We compute out-of-sample fitted values for the 10-year Treasury yield during the subsequent ZLB period and compare it to the actual outcome for the yield, based on the regression results reported in Table 2.1.
Figure 2.2 shows that both specifications suggest a yield that is considerably higher than the actual yield during the ZLB. Adding Treasury supply to the equation slightly lowers the error and brings down the fitted value.

The deviation of the fitted value from the realized yield during this period remains large, suggesting that factors not contained in these regressions were depressing long-term yields when short-term yields hit the ZLB.\(^8\)

We then include the most recent data for the ZLB period, and run the same specification on the entire sample through January 2011, reported in the first column of Table 2.2. The parameter estimates generally change. The expected change in short-term interest rates becomes highly significant and carries the expected sign. Non-borrowed reserves are added to the regression in the second column of Table 2.2. The parameter estimates of other control variables now return to values and significance levels found for the pre-crisis sample, suggesting that it is important to take into account reserves during the ZLB period. The dummy capturing a shift in the level at the ZLB is negative, but not significant. The parameter estimates of all other control variables, except for realized volatility and inflation disagreement, are significant. Signs are according to expectation.

Non-borrowed reserves turn out to be highly significant and negative at the ZLB, but not significantly different from zero during normal times.\(^9\) Are the estimated liquidity effects quantitatively meaningful? Table 2.3 shows the implied contributions of each variable to the change in the level of the 10-year Treasury yield between January 2009 and January 2011. The first column reports the change in the respective variable during our ZLB sample, that is, from January 2009 to January 2011. The second column shows the coefficient estimates from the main regression (the second column of Table 2.2). The third column of Table 2.3 shows how much the 10-year yield is estimated to have changed as a result of the change in the respective explanatory variable during our ZLB sample period. In addition

---

\(^8\)This finding does not depend, and is not driven by, the unemployment gap, although a large part of the increase in the fitted value for the interest rate at the ZLB is due to the sharp rise in unemployment.

\(^9\)The results do not depend on the exact sample. When we exclude average maturity, which allows to estimate the regression through April 2011, the main conclusions remain.
to the public Treasury supply, Table 2.3 reports the contributions of the total Treasury supply and the Fed’s purchases of Treasury securities separately.

The results suggest that both the supply effect and the liquidity effect of the Fed’s purchases have reduced long-term yields between January 2009 and January 2011. The point estimate for reserves suggests that the 2.5 percentage points increase in reserves to GDP between January 2009 and January 2011 was associated with a fall in the level of the long yield by roughly 85 basis points.\(^{10}\) The point estimate for the Treasury supply suggests an additional fall in yields of about 19 basis points on account of supply effects. During the ZLB period, the US Treasury issued a considerable amount of new debt with more than one year maturity. In terms of nominal GDP, the total Treasury supply has increased 19 percentage points. The regression results thus suggest that this would have translated into an increase in long-term yields of 85 basis points. The Fed’s purchases of Treasury securities thus only partially alleviated the supply effect of these new issues by approximately one quarter. The combined supply and liquidity effects suggest that the Fed lowered long-term yields by approximately 104 basis points during this time period. These estimates lie at the upper end of the range found in the recent literature. Reviewing recent results, Hamilton and Wu (2010b) summarize that estimated supply effects have ranged between 17 to 48 basis points. This range refers to the Fed’s LSAP of $400 billion. More recent papers find slightly larger effects. D’Amico and King (2011), for example, find an overall effect of the Fed’s $300 billion Treasury purchases of about 50 basis points on the level of the yield curve, and Krishnamurthy and Vissing-Jørgensen (2011) find - through different channels - an overall effect of quantitative easing in excess of 100 basis points.

3.4 Robustness

In theory, liquidity effects should affect the real rate, whereas the nominal part of the yield may pull in the opposite direction due to Fisher effects. The first column of Table 2.4 hence shows the results for the real 10-year interest rate.\(^{11}\)

\(^{10}\)Note that the main part of the increase in reserves took place in the late autumn of 2008. The effect of this increase is not taken into account here.

\(^{11}\)The monthly inflation expectations from the Cleveland Fed are derived from financial market prices and yields. This means that their measurement error is correlated with yields. Therefore, adding inflation
Both the liquidity effect and the supply effect remain highly significant. Assuming that there are both liquidity effects and Fisher effects at work, the liquidity effect should be even stronger for the real yield. However, the estimated size of the liquidity effect is considerably reduced for the real yield, suggesting that inflation expectations did not increase due to higher liquidity.

As argued in Section 2, liquidity effects could potentially affect other parts of the yield curve, as long as yields are positive. To see if this has been the case, we regress the 5-year yield on the explanatory variables in column 1 of Table 2.5. The results suggest that the liquidity effect on the 5-year yield is greater than on the 10-year yield. The second column of Table 2.5, reporting the estimates for the real 5-year yield, shows that liquidity effects are estimated to be smaller for the real yield also at shorter maturity.

As argued in Section 2, the Fed’s purchases of Treasury bonds may have signalled something about the Fed’s future monetary policy intentions, notably how long the Fed intends to keep interest rates low. There is no reason to believe that such signalling effects would be correlated with the times of the Fed’s actual asset purchases (as opposed to the announcement times of these). As a further robustness check, we nevertheless include the two announcement dummies mentioned in Section 3.2. The first column of Table 2.6 shows that a significant announcement effect indeed exists in the data. The Jackson Hole dummy is included in column 2 of Table 2.6, and shows that this was an important date for government bond yields. The liquidity effect is robust to the inclusion of these dummies.

Moreover, there is a potential source of endogeneity in the regressions. While the quantity and time frame for the Fed’s purchases of assets during the LSAPs were determined and

---

expectations as an explanatory variable creates the problem that a large amount of noise enters the yield and inflation index products, which cannot be assigned to changes in expectations or real developments, but rather to liquidity, herding, trading behaviors, etc. Part of this noise enters the expectations measure, depending on how the noise affects yields and prices. The measurement error of an explanatory variable leads to a bias in all parameter estimates. We have no prior of which direction the bias takes. Moreover, if the measurement error is correlated with the dependent variable, the problem of an omitted variable bias arises. Measurement error of a dependent variable, however, does not lead to biased estimates as long as the measurement error is not correlated with the explanatory variables in the regression (see Greene, 1993). Hence, moving the measurement error to the left side of the regression eliminates the problem.
announced in advance, the speed of the Fed’s actual purchases could slightly vary across days, depending on market liquidity. Specifically, the Fed sought to purchase more assets during days of lower market liquidity, where yields tended to be slightly higher. Conversely, relatively less assets were bought on days with high market liquidity. If the influence of market liquidity on the Fed’s asset purchases has been sufficiently strong, we might expect changes in reserves to be related to higher interest rate levels, and this could bias the parameter estimate on reserves downward. In line with the empirical literature on the effectiveness of foreign exchange interventions, Hamilton and Wu (2010) address similar endogeneity problems by relating a forecast of the Treasury yield to changes in Treasury supply. As we are mainly concerned with qualitative results, we note that this source of endogeneity would tend to bias the parameter estimate for reserves downward, suggesting that the parameter estimate is on the conservative side.

Finally, and most importantly, a concern with the regression reported in Table 2.2 is the high persistence and near-unit root dynamics of the dependent and some of the explanatory variables. This raises the question of whether the significant correlations represent causality or are simply spurious. As a first pass at addressing this issue, we use the spread between the 10-year Treasury yield and the 3-month Libor (henceforth the term spread) as dependent variable. In contrast to the 10-year yield, the term spread does not exhibit a trend over the sample period (see 2.5), and while still persistent, it can be accepted borderline stationary depending on the type of unit-root test used. The concern about spurious results is hence mitigated, but not eliminated. The second column in Table 2.4 presents the regression results for the term spread. Although smaller in size, there is still a significantly negative liquidity effect during the ZLB period. Moreover, when using the term spread between the 5-year and the 3-month interest rates, the coefficient is larger in absolute terms and significant at the 1% level. From January 2009 to January 2011, the 3-month Treasury yield increased by 3.5 basis points. Hence, the estimation of liquidity effects on the term spread suggests that liquidity brought down the 10-year yield by 46 basis points during the period in question.

Simply running the specification used in the paper in first differences leads to the significantly negative parameter estimates only when the December 2008 is included in the ZLB
sample, and insignificant estimates otherwise. Thus, we cannot conclude that the estimated empirical association between reserves and yields represents causality in the strict sense. However, for the first differences to yield significant results, it is necessary to specify the dynamics right. We would need to specify how long it takes for changes in reserves to lead to changes in interest rates, and how long we should expect such interest rate effects to last. Neither the theoretical nor the empirical literature offers clear guidance as to what these time lags and persistence should be. Christiano and Eichenbaum (1992) find liquidity effects in the quarter after the expansion takes place, and lasting for nearly 20 quarters, while more recent papers find very short-term liquidity effects of changes in reserves (see for example Seth and Carpenter, 2006). Furthermore, it is reasonable to believe that the time it takes reserves to affect interest rates, and the persistence of the effect, could be variable and perhaps endogenous to economic circumstances. However, the short time period at the ZLB does not permit us to estimate a VAR in first differences with a substantial number of lags to capture such variable liquidity effects.

Concluding on robustness, the evidence is not conclusive and the risk of spurious correlations remains. But the data does not reject that liquidity effects have been at play as a transmission mechanism of the Fed’s asset purchases on long term interest rates. Long term interest rates were lower than what the portfolio balance effects of changes in the supply of Treasury bonds alone can account for, and the correlation of the levels of interest rates with the level of reserves during the ZLB period suggests that liquidity effects could have been a possible cause of the lower level of the long-term interest rate observed at the ZLB. However, more data are needed to establish whether causality in fact runs from reserves to interest rates.

4 Conclusion

This paper argues that at the ZLB, the large scale asset purchases carried out by the Federal Reserve may have had liquidity effects due to increases in reserves as well as portfolio balance effects of the changes in the supply of Treasuries outstanding to the public. Failing to take into account such liquidity effects could lead to an underestimation of the impact
of the large scale asset purchase programs on long-term government bond yields. While correlation is not causality, preliminary evidence suggests that reserves and yields were indeed correlated during the ZLB period, and that this correlation points to economically important effects. Thus, liquidity effects may have reduced long-term yields by 46 to 85 basis points due to the increase in reserves of about 2.5 percentage points of GDP between January 2009 and January 2011.

One upshot of these results is that when liquidity is drained from the banking system in the future, this could lead to a more important increase in long-term yields than expected if only supply effects are considered.
A Figures

Figure 2.1: Non-Borrowed Reserves and Long Term Yields at the ZLB, Quarterly, 2009-2011
The fitted values in the upper and the lower panel correspond to the coefficient estimates in the first and second column of Table 2.1, respectively. The actual 10-year Treasury yield is represented by the solid black line. The fitted values are computed using the estimates of the respective sample size as indicated in each graph. The red dashed line indicates the fitted values within the sample of the estimates. The fat red dashed line depicts the fitted values for the period after the estimation sample ends.
Figure 2.3: Dependent Variables
Figure 2.4: Regressors - Part I

- Federal Funds Target Rate
- Expected Change in 1-Year Rate
- Unemployment Gap
- core CPI Inflation
- Inflation Expectations Uncertainty
- realized Volatility
Figure 2.5: Regressors - Part II

- **average Maturity of Treasury Supply**
- **Public Treasury Supply**
- **non-borrowed Reserves**
### Table 2.1: Pre-Crisis Regression Results

<table>
<thead>
<tr>
<th>Dependent Variable: ( i_t^{(10y)} )</th>
<th>1990/02 to 2008/06</th>
<th>1990/02 to 2008/06</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>( t )-stat</td>
</tr>
<tr>
<td>-------------------------------------</td>
<td>-------------</td>
<td>--------------</td>
</tr>
<tr>
<td>( c )</td>
<td>-0.810*</td>
<td>-1.661</td>
</tr>
<tr>
<td>FFTR</td>
<td>0.579***</td>
<td>9.346</td>
</tr>
<tr>
<td>Expected Change in 1-Year Rate</td>
<td>-0.069</td>
<td>-0.316</td>
</tr>
<tr>
<td>Unemployment Gap</td>
<td>0.914***</td>
<td>7.765</td>
</tr>
<tr>
<td>Core CPI Inflation</td>
<td>0.545</td>
<td>1.622</td>
</tr>
<tr>
<td>Inflation Disagreement</td>
<td>0.065</td>
<td>0.479</td>
</tr>
<tr>
<td>Realized Volatility</td>
<td>0.127</td>
<td>0.223</td>
</tr>
<tr>
<td>Average Maturity</td>
<td>0.012***</td>
<td>7.132</td>
</tr>
<tr>
<td>Treasury Supply</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Adjusted R(^2)</td>
<td>0.8534</td>
<td></td>
</tr>
<tr>
<td>Number of Obs</td>
<td>961</td>
<td></td>
</tr>
</tbody>
</table>

Both reserves and the Treasury supply are measured in percent of GDP.

Newey West standard errors (12 lags). ***, **, * denote significance at the 1, 5, and 10% levels.
Table 2.2: Regression Results including non-borrowed Reserves

<table>
<thead>
<tr>
<th>Sample</th>
<th>Coefficient</th>
<th>t-stat</th>
<th>Coefficient</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-3.183***</td>
<td>-6.873</td>
<td>-1.515***</td>
<td>-3.084</td>
</tr>
<tr>
<td>FFTR</td>
<td>0.412***</td>
<td>6.431</td>
<td>0.522***</td>
<td>8.192</td>
</tr>
<tr>
<td>Expected Change in 1-Year Rate</td>
<td>1.101***</td>
<td>4.385</td>
<td>0.483**</td>
<td>2.105</td>
</tr>
<tr>
<td>Unemployment Gap</td>
<td>-0.111</td>
<td>-1.049</td>
<td>0.527***</td>
<td>4.296</td>
</tr>
<tr>
<td>Core CPI Inflation</td>
<td>1.271***</td>
<td>4.740</td>
<td>0.707**</td>
<td>2.473</td>
</tr>
<tr>
<td>Inflation Disagreement</td>
<td>0.468***</td>
<td>3.202</td>
<td>0.205</td>
<td>1.583</td>
</tr>
<tr>
<td>Realized Volatility</td>
<td>-0.301</td>
<td>-0.659</td>
<td>-0.182</td>
<td>-0.357</td>
</tr>
<tr>
<td>Average Maturity</td>
<td>0.012***</td>
<td>6.539</td>
<td>0.011***</td>
<td>5.819</td>
</tr>
<tr>
<td>Treasury Supply</td>
<td>0.075***</td>
<td>3.279</td>
<td>0.044**</td>
<td>2.265</td>
</tr>
<tr>
<td>Reserves (D_{t}^{pre-ZLB})</td>
<td>-</td>
<td>-</td>
<td>0.078</td>
<td>0.419</td>
</tr>
<tr>
<td>Reserves (D_{t}^{ZLB})</td>
<td>-</td>
<td>-</td>
<td>-0.341***</td>
<td>-2.731</td>
</tr>
<tr>
<td>(D_{t}^{ZLB})</td>
<td>-</td>
<td>-</td>
<td>0.005</td>
<td>0.108</td>
</tr>
<tr>
<td>(D_{t}^{Lehman})</td>
<td>-</td>
<td>-</td>
<td>-0.280</td>
<td>-1.110</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.8604</td>
<td></td>
<td>0.8888</td>
<td></td>
</tr>
<tr>
<td>Number of Obs</td>
<td>1096</td>
<td></td>
<td>1096</td>
<td></td>
</tr>
</tbody>
</table>

Both reserves and the Treasury supply are measured in percent of GDP.

Newey West standard errors (12 lags). ***, **, * denote significance at the 1, 5, and 10% levels;
\(D_{t}^{pre-ZLB}\) up to Dec-2008; \(D_{t}^{ZLB}\) Jan-2009 to end; \(D_{t}^{Lehman}\) Jun-2008 to Dec-2008
Table 2.3: Contributions to the Change in Yield from Jan-2009 to Jan-2011

<table>
<thead>
<tr>
<th></th>
<th>2009/01 to 2011/01</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Change Estimated</td>
</tr>
<tr>
<td></td>
<td>Coefficient</td>
</tr>
<tr>
<td>$t_1^{(10y)}$</td>
<td>1.201 - -</td>
</tr>
<tr>
<td>FFTR</td>
<td>0.000 0.522 0.000</td>
</tr>
<tr>
<td>Expected Change in 1-Year Rate</td>
<td>0.571 0.483 0.276</td>
</tr>
<tr>
<td>Unemployment Gap</td>
<td>1.620 0.527 0.853</td>
</tr>
<tr>
<td>Core CPI Inflation</td>
<td>0.238 0.707 0.168</td>
</tr>
<tr>
<td>Inflation Disagreement</td>
<td>0.100 0.205 0.021</td>
</tr>
<tr>
<td>Realized Volatility</td>
<td>-0.225 -0.182 0.041</td>
</tr>
<tr>
<td>Average Maturity</td>
<td>46.248 0.011 0.487</td>
</tr>
<tr>
<td>Public Supply</td>
<td>14.591 0.044 0.646</td>
</tr>
<tr>
<td>Total Supply</td>
<td>18.975 0.044 0.840</td>
</tr>
<tr>
<td>Fed Holdings</td>
<td>4.384 -0.044 -0.194</td>
</tr>
<tr>
<td>Reserves</td>
<td>2.489 -0.341 -0.849</td>
</tr>
</tbody>
</table>

The coefficients correspond to the last column in Table 2.2. Both the supply measures and reserves are measured in percent of GDP.
Table 2.4: Robustness Check I: Real Yield and Term Spread

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>( i_{t}^{(10y)} - E_t[\pi^{(10y)}] )</th>
<th>( i_{t}^{(10y)} - i_{t}^{(3m)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>Coefficient</td>
<td>Coefficient</td>
</tr>
<tr>
<td></td>
<td>02/1990 to 01/2011</td>
<td>02/1990 to 01/2011</td>
</tr>
<tr>
<td>c</td>
<td>-1.752***</td>
<td>-1.348***</td>
</tr>
<tr>
<td>FFTR</td>
<td>0.322***</td>
<td>-0.389***</td>
</tr>
<tr>
<td></td>
<td>7.502</td>
<td>6.973</td>
</tr>
<tr>
<td>Expected Change in 1-Year Rate</td>
<td>0.452***</td>
<td>0.826***</td>
</tr>
<tr>
<td>Unemployment Gap</td>
<td>0.262***</td>
<td>0.373***</td>
</tr>
<tr>
<td>Core CPI Inflation</td>
<td>0.498**</td>
<td>0.646***</td>
</tr>
<tr>
<td>Inflation Disagreement</td>
<td>0.162*</td>
<td>0.243**</td>
</tr>
<tr>
<td>Realized Volatility</td>
<td>0.023</td>
<td>0.190</td>
</tr>
<tr>
<td>Average Maturity</td>
<td>0.006***</td>
<td>0.009***</td>
</tr>
<tr>
<td>Treasury Supply</td>
<td>0.034***</td>
<td>0.035**</td>
</tr>
<tr>
<td>Reserves (D_{t}^{pre-ZLB})</td>
<td>0.039</td>
<td>-0.052</td>
</tr>
<tr>
<td>Reserves (D_{t}^{ZLB})</td>
<td>-0.178**</td>
<td>-0.183*</td>
</tr>
<tr>
<td>(D_{t}^{ZLB})</td>
<td>-0.001</td>
<td>-0.370</td>
</tr>
<tr>
<td>(D_{t}^{Lehman})</td>
<td>-0.226</td>
<td>0.629***</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.8523</td>
<td>0.8871</td>
</tr>
<tr>
<td>Number of Obs</td>
<td>1096</td>
<td>1096</td>
</tr>
</tbody>
</table>

Both reserves and the Treasury supply are measured in percent of GDP.
Newey West standard errors (12 lags). ***., **. *, * denote significance at the 1, 5, and 10% levels;
\(D_{t}^{pre-ZLB}\) up to Dec-2008; \(D_{t}^{ZLB}\) Jan-2009 to end; \(D_{t}^{Lehman}\) Jun-2008 to Dec-2008
Table 2.5: Robustness Check II: 5-Year Treasury Yield

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>( i_t^{(5y)} )</th>
<th>( i_t^{(5y)} - E_t[\pi_t^{(5y)}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>Coefficient</td>
<td>Coefficient</td>
</tr>
<tr>
<td></td>
<td>02/1990 to 01/2011</td>
<td>02/1990 to 01/2011</td>
</tr>
<tr>
<td>c</td>
<td>-1.629***</td>
<td>-1.640***</td>
</tr>
<tr>
<td></td>
<td>-3.219</td>
<td>-4.590</td>
</tr>
<tr>
<td>FFTR</td>
<td>0.616***</td>
<td>0.375***</td>
</tr>
<tr>
<td></td>
<td>9.175</td>
<td>8.133</td>
</tr>
<tr>
<td>Expected Change in 1-Year Rate</td>
<td>0.059</td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td>0.229</td>
<td>0.141</td>
</tr>
<tr>
<td>Unemployment Gap</td>
<td>0.483***</td>
<td>0.182*</td>
</tr>
<tr>
<td></td>
<td>3.608</td>
<td>1.426</td>
</tr>
<tr>
<td>Core CPI Inflation</td>
<td>0.560*</td>
<td>0.331</td>
</tr>
<tr>
<td></td>
<td>1.782</td>
<td>0.515</td>
</tr>
<tr>
<td>Inflation Disagreement</td>
<td>0.118</td>
<td>0.052</td>
</tr>
<tr>
<td></td>
<td>0.855</td>
<td>0.858</td>
</tr>
<tr>
<td>Realized Volatility</td>
<td>-0.521</td>
<td>-0.322</td>
</tr>
<tr>
<td></td>
<td>-0.912</td>
<td>-0.816</td>
</tr>
<tr>
<td>Average Maturity</td>
<td>0.010***</td>
<td>0.005***</td>
</tr>
<tr>
<td></td>
<td>4.934</td>
<td>3.582</td>
</tr>
<tr>
<td>Treasury Supply</td>
<td>0.042*</td>
<td>0.034**</td>
</tr>
<tr>
<td></td>
<td>1.958</td>
<td>2.364</td>
</tr>
<tr>
<td>Reserves - ( D_t^{pre-ZLB} )</td>
<td>0.093</td>
<td>0.080</td>
</tr>
<tr>
<td></td>
<td>0.530</td>
<td>0.658</td>
</tr>
<tr>
<td>Reserves - ( D_t^{ZLB} )</td>
<td>-0.453***</td>
<td>-0.313***</td>
</tr>
<tr>
<td></td>
<td>-3.285</td>
<td>-3.384</td>
</tr>
<tr>
<td>( D_t^{ZLB} )</td>
<td>0.691</td>
<td>0.952*</td>
</tr>
<tr>
<td></td>
<td>0.879</td>
<td>1.791</td>
</tr>
<tr>
<td>( D_t^{Lehman} )</td>
<td>-0.535**</td>
<td>-0.443**</td>
</tr>
<tr>
<td></td>
<td>-2.062</td>
<td>-2.487</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.9040</td>
<td>0.8753</td>
</tr>
<tr>
<td>Number of Obs</td>
<td>1096</td>
<td>1096</td>
</tr>
</tbody>
</table>

Both reserves and the Treasury supply are measured in percent of GDP.
Newey West standard errors (12 lags). ***, **, * denote significance at the 1, 5, and 10% levels;
\( D_t^{pre-ZLB} \) up to Dec-2008; \( D_t^{ZLB} \) Jan-2009 to end; \( D_t^{Lehman} \) Jun-2008 to Dec-2008
Table 2.6: Robustness Check III: Announcement Effects

<table>
<thead>
<tr>
<th>Sample</th>
<th>1990/02 to 2011/01</th>
<th>1990/02 to 2011/01</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>t-stat</td>
</tr>
<tr>
<td>c</td>
<td>-1.500***</td>
<td>-3.052</td>
</tr>
<tr>
<td>FFTR</td>
<td>0.519***</td>
<td>8.184</td>
</tr>
<tr>
<td>Expected Change in 1-Year Rate</td>
<td>0.460**</td>
<td>2.035</td>
</tr>
<tr>
<td>Unemployment Gap</td>
<td>0.529***</td>
<td>4.315</td>
</tr>
<tr>
<td>Core CPI Inflation</td>
<td>0.699**</td>
<td>2.449</td>
</tr>
<tr>
<td>Inflation Disagreement</td>
<td>0.207</td>
<td>1.593</td>
</tr>
<tr>
<td>Realized Volatility</td>
<td>-0.157</td>
<td>-0.311</td>
</tr>
<tr>
<td>Average Maturity</td>
<td>0.011***</td>
<td>5.849</td>
</tr>
<tr>
<td>Event Dummy</td>
<td>-0.369**</td>
<td>-2.547</td>
</tr>
<tr>
<td>Jackson Hole Dummy</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Treasury Supply</td>
<td>0.044**</td>
<td>2.234</td>
</tr>
<tr>
<td>Reserves - $D_t^{pre-ZLB}$</td>
<td>0.107</td>
<td>0.567</td>
</tr>
<tr>
<td>Reserves - $D_t^{ZLB}$</td>
<td>-0.348***</td>
<td>-2.837</td>
</tr>
<tr>
<td>$D_t^{ZLB}$</td>
<td>0.078</td>
<td>0.110</td>
</tr>
<tr>
<td>$D_{Lehman}$</td>
<td>-0.240</td>
<td>-0.973</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.8893</td>
<td></td>
</tr>
<tr>
<td>Number of Obs</td>
<td>1096</td>
<td></td>
</tr>
</tbody>
</table>

Both reserves and the Treasury supply are measured in percent of GDP.
Newey West standard errors (12 lags). ***, **, * denote significance at the 1, 5, and 10% levels;
$D_t^{pre-ZLB}$ up to Dec-2008; $D_t^{ZLB}$ Jan-2009 to end; $D_{Lehman}$ Jun-2008 to Dec-2008
C The Data

The analysis uses weekly data. Both the data on non-borrowed reserves and on the Fed’s holdings of Treasury securities are available as end-of-Wednesday levels. We hence define the beginning of a week as Thursday and the end as Wednesday. For the data available on a daily basis, we compute one-week averages of the daily data on a weekly frequency. Thus, given $t - 1$ corresponds to a Wednesday, then

$$i_t = \frac{1}{5} \sum_{s=0}^{4} i_{t+s} \text{ for } \{i_t, i_{t+5}, \ldots\}$$

(2.2)

The data available only at a lower frequency are linearly interpolated, matching the monthly observation with the last week’s observation of the month. Accordingly, Figures 2.3 to 2.5 depict the final data entering the regressions.

The 5-year and 10-year Treasury bond yields are from Datastream. Real 5-year and 10-year yields are constructed using the Fisher equation and data on inflation expectations at the two horizons from the Federal Reserve Bank of Cleveland.\(^\text{12}\)

The Federal Funds target rate (FFTR) is retrieved from the FRED (DFEDTAR). It starts in 1982, but is discontinued after 2008. We therefore link it with recent data from Bloomberg.

Core CPI inflation is retrieved from the BIS. The unemployment gap is measured as the difference between the monthly unemployment rate retrieved from Datastream and the Congressional Budget Office’s estimate of the NAIRU.\(^\text{13}\) As in Gagnon et al. (2010), long-run inflation disagreement is measured as the interquartile range of 5- to 10-year ahead inflation expectations, as reported by the Michigan Survey of Consumers.\(^\text{14}\) The six-month

\(^\text{12}\)The Cleveland Fed’s data on inflation expectations can be downloaded from http://www.clevelandfed.org/research/data/inflation_expectations/index.cfm.

\(^\text{13}\)The CBO’s estimate of the NAIRU can be downloaded from http://www.cbo.gov/doc.cfm?index=12039.

\(^\text{14}\)This data can be downloaded from http://www.sca.isr.umich.edu/main.php.
realized daily volatility is computed using a rolling 24-week window of the 10-year Treasury yield.

Gagnon et al. (2010) use the Eurodollar slope as a proxy for interest rate expectations. They use the difference between the implied rates on Eurodollar futures contracts settling approximately two-years and one-year ahead. We approximate the 2-year-ahead Eurodollar rate by the implied 1-year forward rate from 2-year and 3-year Treasuries. Similarly, we approximate the 1-year-ahead Eurodollar rate by the implied 1-year forward rate from 1-year and 2-year Treasury yields. The expectations hypothesis defines the 2-year and the 3-year Treasury yields as follows.

\[ 1 + i_t^{(2y)} = \left( 1 + i_t^{(1y)} \right) \left( 1 + E_t^{i_t^{(1y)}} \right) \]  
(2.3)

\[ 1 + i_t^{(3y)} = \left( 1 + i_t^{(2y)} \right) \left( 1 + E_t^{i_t^{(1y)}} \right) \]  
(2.4)

The expected 1-year rate one year and two years ahead are thus

\[ E_t^{i_t^{(1y)}} = \frac{1 + i_t^{(2y)}}{1 + i_t^{(1y)}} - 1, \quad \text{and} \]
\[ E_t^{i_t^{(3y)}} = \frac{1 + i_t^{(3y)}}{1 + i_t^{(2y)}} - 1. \]  
(2.5)

To get the expected change of the 1-year rate one year ahead, we use the difference between the two, i.e. \( E_t^{i_t^{(1y)}} - E_t^{i_t^{(1y)}} \).

For the Treasury supply, we use the data provided by Hamilton and Wu (2010b) as well as data directly retrieved from the SOMA. Hamilton and Wu provide data on total Treasury supply and on Treasury supply to the public, calculated as total supply minus Fed holdings. The periodicity of their data is monthly. Since the SOMA provides weekly data as of Wednesdays, we compute the public supply ourselves. Unfortunately, the data from

15Hamilton and Wu (2010b) provide their data on the Treasury supply from \text{http://econ.ucsd.edu/~jingwu/zlb_data.html}.

16The data on the Factors Affecting Reserve Balances (H.4.1) is published by the Board of Governors of the Federal Reserve System and can be downloaded from \text{http://www.federalreserve.gov/datadownload/Choose.aspx?rel=H41, Table 2}.
### Table 2.7: Events with Potential Announcement Effects

<table>
<thead>
<tr>
<th>Date</th>
<th>Event Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 Nov 2008</td>
<td>initial LSAP announcement</td>
</tr>
<tr>
<td>01 Dec 2008</td>
<td>Bernanke’s announcement of possible purchase of long-term Treasury securities</td>
</tr>
<tr>
<td>16 Dec 2008</td>
<td>FOMC statement: expansion of LSAPs</td>
</tr>
<tr>
<td>28 Jan 2009</td>
<td>FOMC statement: expansion of LSAPs</td>
</tr>
<tr>
<td>18 Mar 2009</td>
<td>FOMC statement: purchases of “up to” $300 billion of long-term Treasury securities</td>
</tr>
<tr>
<td>12 Aug 2009</td>
<td>FOMC statement: drop of “up to” language; gradual slowing or purchases</td>
</tr>
<tr>
<td>23 Sep 2009</td>
<td>FOMC statement: drop of “up to” language; gradual slowing or purchases</td>
</tr>
<tr>
<td>04 Nov 2009</td>
<td>FOMC statement: purchase of around $175 billion of agency debt</td>
</tr>
<tr>
<td>27 Aug 2010</td>
<td>Bernanke’s speech at Jackson Hole</td>
</tr>
<tr>
<td>10 Aug 2010</td>
<td>FOMC statement: Reinvestment to keep reserve balances steady</td>
</tr>
<tr>
<td>21 Sep 2010</td>
<td>FOMC statement: keep policy of reinvesting</td>
</tr>
<tr>
<td>03 Nov 2010</td>
<td>FOMC statement: keep reinvesting and expanding purchases of long-term Treasury securities</td>
</tr>
</tbody>
</table>

The SOMA starts only as of end 2002. Therefore, we extend the public Treasury supply series backwards by linking it with the total Supply, which starts in February 1990. Treasury supply enters the regressions in percent of nominal GDP. The average maturity of total outstanding Treasury bonds is also provided by Hamilton and Wu (2010b). It states the average maturity of debt held by the public, measured in weeks.

Weekly non-borrowed reserves are computed by subtracting total borrowings (TOTBORR) from total reserves held with the Fed (WRESBAL). Both are published by the FRED. In line with the Treasury supply, reserves enter the regressions in percent of nominal GDP.

Business cycle measures are taken from different sources. First, the unemployment gap [fill in]. The real growth rate of output (Gross Domestic Production) is from the International Financial Statistics database of the IMF. The output gap is taken from the BIS. Finally, the dummy variable of events controls for announcement effects. The dates listed in Table 2.7 are included. A more detailed description of these events is given in Gagnon et al. (2010) and Neely (2010).
Chapter 3

Central Bank Reserves and the Yield Curve at the ZLB

Abstract

With short-term interest rates bounded at zero, monetary policy has aimed at affecting the yield curve at the longer end during the recent years. As the recent literature has shown, the quantitative easing programs conducted by the Federal Reserve have significantly lowered long-term yields. This paper adds central bank reserves as a fourth factor to an affine term structure model to estimate the effect of quantitative easing on the yield curve. The cumulative effect on 10-year Treasury securities during the zero lower bound period is estimated to amount to 85 basis points. Of the total effect, one quarter is shown to be due to the liquidity effect and three quarters to the supply effect. To disentangle the two effects, the estimates for the US are compared to estimates for Swiss data because the Swiss national bank did not engage in any government bond purchases.
1 Introduction

Central banks around the globe have been flooding financial markets with liquidity since the breakdown of interbank markets in response to the collapse of Lehman Brothers in the fall of 2008. In its initial phase of unconventional monetary policy, also referred to as QE1, the US Federal Reserve performed large scale asset purchases (LSAP) consisting mainly of mortgage-backed securities. The Federal Reserve then gradually extended liquidity by outright purchases of Treasury securities. When the Treasury purchase program known as QE2 came to an end in mid 2011, non-borrowed reserves (NBR) held at the Federal Reserve exceeded USD 1.6 Trillion.

Aside from signalling effects about future policy, the literature has focussed mainly on portfolio balance effects of outright Treasury securities’ purchases of the Federal Reserve\(^1\), in particular the so-called supply effect and spill over effects from outright purchases of similar assets such as mortgage backed securities and agency debt\(^2\). Krogstrup et al. (2012) have shown that, in addition to the supply effect, there is also a liquidity effect of quantitative easing (QE) that has affected long-term Treasury bonds. While the supply effect depends on the composition of the central bank’s balance sheet, it is an expansion of the balance sheet that triggers the liquidity effect.

This study proceeds in two steps: it first estimates the total effect of QE on term premia in an Affine Term Structure Model (ATSM) framework; in a second step it attempts to disentangle the supply from the liquidity effect. Our estimates show that QE has significantly lowered the term premium of Treasury yields. Estimates of the cumulative effect of QE on 10-year Treasury yields during the zero lower bound (ZLB) amounts to 85 basis points. This result is in line with the recent literature. Estimates of Swiss data suggest 10-year yields decreased 34 basis points in response to the expansion in liquidity. Because the Swiss National Bank (SNB) has not engaged in government bond purchases during the ZLB, the Swiss estimates can be fully ascribed to the liquidity effect. In the US, approximately 21

\(^1\)See e.g. Gagnon et al. (2011), Hamilton and Wu (2010b), Krishnamurthy and Vissing-Jorgensen (2011)

\(^2\)See e.g. D’Amico and King (2010)
basis points can be attributed to the liquidity effect and 64 basis points to the supply effect.

QE may affect either the expectations’ part of interest rates or the term premium. While signalling effects change expectations about future policy, the supply and the liquidity effect operate on the term premium. Term structure models are suitable to isolate the term premium. The three latent factors are typically shown to account for changes in expected future interest rates and thus capture any signalling effects and announcement effects that affect expectations about the future policy rate. Christensen and Rudebusch (2012) isolate the term premium in an affine term structure model and find that in US data, most of the effect of QE on yields was transmitted through policy expectations rather than the term premium. Hence, they find that the portfolio balance effect - or the supply effect - of QE on yields was much smaller than the signalling effect in the US. Information theory would suggest that the supply effect materializes immediately after announcements of the asset purchases. Therefore, many papers\(^3\) apply event studies to estimate the effects. Instead, we add a fourth factor to the term structure model, a measure for liquidity, to explain changes in the term premium. The main analysis is computed for US data, divided into two subsamples: a pre-crisis sample that should account for “normal” times and a ZLB sample. To better separate the supply effect from the liquidity effect, we additionally run the analysis for Swiss data as a counterfactual case. The injected liquidity by the SNB was entirely driven by foreign exchange purchases rather than government bonds. Hence, a supply effect can be excluded.

The rest of the paper is organized as follows. Section 2 lays out the different effects of quantitative easing on yields. Section 3 illustrates the US and Swiss data sets. Section 4 introduces the term-structure model used and Section 5 provides the details on the model estimation. Finally, Section 6 presents the results and Section 7 concludes.

\(^3\)See for example Gagnon et al. (2010) and Bauer and Rudebusch (2011).
2 The Different Effects of QE on Interest Rates

The recent literature has focused on two effects of the Federal Reserves’ asset purchases. First, measures undertaken by the monetary authority are typically thought to have a signaling effect about future monetary policy. For example, Christensen and Rudebusch (2012) find that most of the declines in US interest rates can be attributed to lower policy expectations. Second, the purchase of long-term assets changes the maturity structure of the assets available to the public which might trigger a portfolio balance effect. Accordingly, Gagnon et al. (2011) attribute most of the effect of the LSAP on long-term yields to this portfolio balance effect, the so-called supply effect. Krogstrup et al. (2012) suggest a third mechanism at work: a liquidity effect on longer-term yields. Conversely to the first two effects, the liquidity effect hinges on the expansion rather than the composition of a central banks’ balance sheet. By augmenting its balance sheet, the central bank creates additional funding and provides it to the economy, so there is more funding available to finance an unchanged amount of debt. Krogstrup et al. (2012) argue that some of the additional credit might have found its way into Treasury bonds and thereby brought down long-term yields.

The signalling effect can be clearly distinguished from the other two as it affects the expectations’ part of yields rather than the term premium. It is not straightforward, however, to disentangle the liquidity effect from the supply effect. With an outright purchase of a Treasury bond, the Federal Reserve temporarily raises demand and therefore the price of that bond. Since the bond is drawn from the market, the supply of that bond decreases. Given unchanged market demand for this specific bond, the price increases and the yield decreases permanently. This effect is referred to as the supply effect in the literature.\(^4\)

The supply effect boils down to a portfolio balance effect that arises if there is some kind of “preferred-habitat” behavior of investors, e.g. if investors have a preference for the specific maturity of the asset purchased by the central bank. Some research suggests that this portfolio balance effect might even be at work across different asset classes. D’Amico and King (2010), for example, suggest that the purchases of mortgage backed securities

conducted by the Fed significantly affected long-term Treasury yields. In addition to the supply effect, the price change will spread to close-by maturities due to arbitrage. That said, an outright purchase of an asset conducted by the central bank raises the amount of credit in the form of reserves available in the aggregate. Assuming an unchanged amount of government debt, i.e. Treasury bonds, prices of these bonds will, all else equal, increase as there is more money to buy the same amount of assets. This is what Krogstrup et al. (2012) refer to as the liquidity effect. The liquidity effect in the sense of Friedman (1968) refers to a fall in short-term nominal interest rates after an exogenous persistent increase in narrow measures of the money supply. When the policy rate hits the zero lower bound (ZLB), the liquidity effect on the short-term yields is arguably absent, as the short-term Treasury bills and money become equivalents. However, as the banks that sell assets to the central bank hold more money after the transaction, they might seek a positive return further out the yield curve.

The supply and the liquidity effect are difficult to distinguish empirically. The expansion of the Federal Reserve’s balance sheet was largely due to the purchase of US Treasury bonds. Thus, the change in the supply of bonds, measured as the quantity of these bonds held by the Federal Reserve, is correlated with the amount of liquidity, measured as the deposits banks hold at the Federal Reserve. We address this difficulty by comparing estimates from US data with those from Swiss data. In the case of Switzerland, the supply effect cannot have been at play as the SNB increased liquidity through purchases of foreign exchange rather than government bonds.

5 The amount of money relative to the volume of assets increases, ceteris paribus, against the entire spectrum of assets. So the same price effect might be observed for other assets, too. The effect on Treasury bonds might be particularly large because these assets are very safe and liquid. Krishnamurthy and Vissing-Jorgensen (2011) argue that these characteristics make government bonds constitute a close substitute for money.

6 See e.g. Thornton (2008b).
3 Data

The following section briefly describes and illustrates both the US and the Swiss data sets used and explains how we conduct our analysis in two subsamples, one corresponding to “normal times” and another to the ZLB period.

3.1 US Data

The analysis is conducted on a weekly basis, so the data is converted into the same frequency. The sample spans from June 2005 to October 2011. Because NBR data are published on Wednesdays, all data are snapshots as of that day. Interest-rate data used stem from off-the-run US Treasury Bills and Bonds. In particular, we use 3-month and 6-month secondary market T-bills rates\(^7\) and 1-year to 10-year off-the-run constant maturity yields from Gurkaynak et al. (2007)\(^8\). All the yields are continuously compounded, whereas quarterly and semi-annual compounding is assumed for the 3-month and the 6-month rate, respectively.\(^9\)

In the literature, different measures of money have been used to analyze the liquidity effect. As concluded by Pagan and Robertson (1995), models defining money as narrow measures are more successful in giving evidence for a liquidity effect than those using broader measures of money. This is due to the fact that shocks to broader monetary aggregates are largely due to shocks in money demand rather than to shocks in money supply.\(^10\) Hence, the expansion in liquidity conducted by the Federal Reserve is measured by the amount of reserves banks hold with the Federal Reserve. Total reserves are often found not to correlate with interest rates because the part of reserves which is borrowed is driven by the bank’s demand for reserves. This makes borrowed reserves endogenous, as the demand for

---

\(^7\)Obtained from the Federal Reserve Economic Data base (FRED), under DTB3 and DTB6. The two are not constant-maturity yields which is acceptable due to their short maturity.

\(^8\)The data can be downloaded from http://www.federalreserve.gov/econresdata/researchdata.htm.


\(^10\)See e.g. Bernanke and Blinder (1992), Sims (1992), or Christiano and Eichenbaum (1995).
liquidity responds to monetary policy changes.\textsuperscript{11} We therefore use NBR, i.e. total reserves minus borrowed reserves,\textsuperscript{12} as the measure of liquidity.

The main hypothesis of the paper is that liquidity effects are present at the longer end of the yield curve only when interest rates are close to the ZLB. Therefore, the analysis is carried out in two subsamples. The first can be characterized as “normal times” with interest rates far away from the ZLB. It is defined as starting with the first week of June 2005 until the end of July 2007.\textsuperscript{13} The second subsample corresponds to the ZLB period. It starts with the second week of December 2008, when the Federal funds rate reached the ZLB, and runs through October 2011, the end of the sample (see Figure 3.1).

Figure 3.1 depicts the data. The NBR fluctuated around the level of 10 billion USD in the first subsample shown in the upper panel. NBR amounted to roughly 540 billion USD at the time the Federal funds rate reached the ZLB in late 2008. Then, during the ZLB period shown in the lower panel, NBR continued to grow to reach 1,560 billion USD by October 2011. In the first and the second subsample, the standard deviation of NBR equals 2.9 and 318.5 billion USD, respectively. In order to make the estimation results from the two periods comparable, the NBR factor is normalized so that it exhibits a standard deviation of one within each subsample.

### 3.2 Swiss Data

The data used in the analysis are the 3-month and the 6-month CHF Libor rates and 1-year to 10-year constant-maturity zero-coupon yields on Swiss Confederation Bonds. The data on giro account balances (i.e. sight deposits) of both domestic and foreign banks at the SNB is used to measure liquidity. The SNB publishes this data on a weekly basis, so

\textsuperscript{11}See e.g. Carpenter and Demiralp (2008).

\textsuperscript{12}Borrowed reserves are equal to the sum of credit extended through the Federal Reserve’s regular discount window programs and credit extended through certain Federal Reserve liquidity facilities. The total reserves (WRESBAL) and the total borrowings (TOTBORG) are also available from the FRED.

\textsuperscript{13}We could have used a longer sample for the non-ZLB period. Trying different sample lengths and start/end points does not change the non-ZLB results.
the analysis can also be conducted at the weekly frequency. In line with the US case, we choose the two subsamples so that one corresponds to “normal times” and the second to the ZLB period. The former subsample starts in the first week of January 2006 and ends on 12 September 2008. The second subsample starts with the second week of December 2008, when the SNB decreased its targeted level for the 3-month Libor from 1% to 0.5%, and ends on 7 October 2011.

### 3.3 Simple Data Inspection

Figure 3.1 presents US data and Figure 3.2 Swiss data, where the upper panels show the earlier subsample and the lower panels depict the ZLB period. It is very difficult to see any comovement between interest rates and NBR in both samples in either data set. To get a first idea on such a comovement, Table 3.1 depicts correlations between different yields and NBR. The numbers are reported for both the US and Switzerland. In the upper panel, the correlation refers to levels, whereas the lower panel computes the correlations in first differences. The simple correlations draw a mixed picture.

Macroeconomic theory holds that, in normal times, the liquidity effect dominates at the short end of the yield curve, while at the longer end the Fisher effect is thought to obscure the liquidity effect. Hence, we would expect the correlation between NBR and short-term yields to be negative, while the correlation with long-term yields would turn positive with rising inflation expectations. As Table 3.1 shows in the first row, the data does provide evidence for the liquidity effect as correlations are negative in the US data. The negative correlation weakens with maturity, however, suggesting that the dominance of the Fisher effect increases as we move out on the yield curve. The Swiss data provides the exact opposite picture; there is no evidence for a liquidity effect, and the Fisher effect seems to decrease with maturity. Correlations decrease considerably during the ZLB period for both countries. In US data the negative correlation is largest around the 1-year maturity while the Swiss data suggests longer maturities to be affected the most.

---

14The exact choice of the “normal times” subsample does not affect the results. Preliminary analysis of several different non-ZLB subsamples showed no liquidity effect irrespective of the exact period.
4 The Model

This section defines the term structure model we use to estimate the liquidity effect. We start from the general asset pricing equation, define the pricing kernel and specify a Gaussian diffusion process of the underlying state factors. Then, we specify the one-period interest rate and formulate the bond prices across the maturity spectrum.

4.1 General Setting and State Dynamics

The general asset pricing equation\(^{15}\) under the physical probability measure \(\mathbb{P}\) reads

\[
P_{n,t} = E_t [M_{t+1} P_{n-1,t+1} | I_t],
\]

where \(P_{n,t}\) is the price of an \(n\)-period to maturity zero-coupon bond at time \(t\), \(M_{t+1}\) denotes the stochastic discount factor, and \(I_t\) represents the agents’ current information set. In a risk-neutral world, where investors request no risk compensation, the price of the bond \(P_{n,t}\) equals

\[
P_{n,t} = E^Q_t [\exp(-y_{1,t}) P_{n-1,t+1} | I_t],
\]

where \(Q\) is the risk-neutral probability measure and \(y_{1,t}\) is the short-term interest rate. The no-arbitrage argument assures that the two prices in (3.1) and (3.2) are equal. There exists an equivalent martingale measure \(Q\) according to which (3.2) holds\(^{16}\) with the stochastic discount factor taking the form

\[
\exp(-y_{1,t}) = E_t [M_{t+1} | I_t] = \exp(-y_{1,t}) E_t [(dQ/d\mathbb{P})_{t+1} | I_t].
\]

\(^{15}\)See Campbell et al. (1997).

\(dQ/dP\) is the Radon-Nykodim derivative\(^{17}\) which follows a log-normal process, so it reads

\[
\frac{dQ}{dP}_{t+1} = \exp\left(\frac{-1}{2}(\lambda_t)'\lambda_t - (\lambda_t)'\varepsilon_{t+1}\right),
\]  

(3.3)

\(\lambda_t\) is the market price of risk associated with the sources of uncertainty \(\varepsilon_t\).\(^{18}\) Following Duffee (2002), the market price of risk is an “essentially affine” function of the state variables \(X_t\), so it can be written as

\[
\lambda_t = \lambda_0 + \lambda_1 X_t.
\]  

(3.4)

Equations (3.3) to (3.4) jointly define the pricing kernel of the model, where the essentially affine market price of risk constitutes the first fundamental building block of the Gaussian term structure model.

Another fundamental building block of the Gaussian term structure model is the multivariate state variable \(X_t\). It follows a discrete version of the constant volatility Ornstein-Uhlenbeck process.\(^{19}\) Under the physical probability measure \(P\), the process is

\[
X_{t+1} = (I - \Psi)\mu + \Psi X_t + \Sigma\varepsilon_{t+1}.
\]  

(3.5)

The first term on the right-hand side of equation (3.5) is a vector of the factors’ means. \(\Psi\) is the VAR matrix and \(\Sigma\) is the covariance matrix that normalizes the residuals \(\varepsilon_t\) which are assumed to be standard normal \(i.i.d.\) shocks.

### 4.2 Short Rate and Bond Prices

Following Duffie and Kan (1996), the one-period interest rate

\[
y_{1,t} = A_1 + B_1 X_t
\]

is an affine function of risk factors \(X_t\), where the coefficient \(A_1\) corresponds to the average

\(^{17}\)See Dai et al. (2007).

\(^{18}\)See Ang and Bekaert (2002) and Ang and Piazzesi (2003).

\(^{19}\)See Phillips (1972).
one-period rate in the sample and $B_1$ is a vector of loadings of the risk factors on $y_{1,t}$. The risk factors are

$$X_t = \begin{bmatrix} l_{1,t} \\ l_{2,t} \\ l_{3,t} \\ R_t \end{bmatrix}$$

(3.6)

The first three factors $l_i^t$, $i = 1, 2, 3$ denote the latent factors backed out from yields, $R_t$ denotes central bank reserves. As commonly in the term structure literature, these factors can be interpreted as a level, a slope, and a curvature factor.\(^{20}\) In order to estimate the liquidity effect on the longer end of the yield curve, this paper adds central bank reserves as a fourth factor to $X_t$.\(^{21}\) As it can be noticed, the model assumes no ZLB for the short rate. Yet, as shown in the results section, the model fits the data reasonably well and thus can be considered as a good approximation.

Assuming joint log-normality of bond prices and the pricing kernel in equation (3.1), the $n$-periods to maturity nominal bond price is an affine function of the state variables\(^ {22}\) and thus takes the form

$$p_{n,t} = -A_n - B_n X_t$$

(3.7)

with

$$A_n = A_{n-1} + B_{n-1} ((I - \Psi)\mu - \Sigma \lambda_0) + \frac{1}{2} B_{n-1} \Sigma \Sigma' B_{n-1} + A_1$$

$$B_n = B_{n-1} (\Psi - \Sigma \lambda_1) + B_1.$$  

(3.8)

\(^{20}\)See Section 5 for more details on “backing out” of the yield-only factors.

\(^{21}\)Ang and Piazzesi (2003), for instance, are among many studies that use macro-economic variables as explicit factors.

\(^{22}\)See for example Cochrane and Piazzesi (2009).
5 Estimation

This section derives the likelihood function used to construct the joint posterior of parameters and data. In addition, the model is estimated with a simple version of the Bayesian Markov-Chain Monte-Carlo (MCMC) method and this section provides the general idea, rationale and the description of the algorithm.

5.1 Likelihood Function

Following Chen and Scott (1993), the 6-month, the 5-year and the 10-year yields are set to be observable, while the rest is measured with an error. Let \( y_{o,t} \) be a vector of observed yields that are perfectly priced by the model. Then, equation (3.7) can be written as

\[
\begin{bmatrix}
y_{o,t} \\
R_t
\end{bmatrix} =
\begin{bmatrix}
A_o & B_o & 0_{3 \times 1} \\
0_{1 \times 3} & 1 & 0_{1 \times 3} \\
\end{bmatrix}
\begin{bmatrix}
X_{o,t} \\
R_t
\end{bmatrix},
\]

where \( A_o \) is a \( 3 \times 1 \) vector and \( B_o \) a \( 3 \times 3 \) matrix of factor loadings. \( X_{o,t} \) contains the three latent factors obtained by inverting the above equation as:

\[
\begin{bmatrix}
X_{o,t} \\
R_t
\end{bmatrix} =
\begin{bmatrix}
B_o & 0_{3 \times 1} \\
0_{1 \times 3} & 1 \\
\end{bmatrix}^{-1}
\begin{bmatrix}
y_{o,t} \\
R_t
\end{bmatrix} -
\begin{bmatrix}
A_o \\
0_{1 \times 1}
\end{bmatrix}.
\]

The first part of the likelihood function refers to the evolution of the state variables \( X_t \) as given in equation (3.5). The assumption that \( \varepsilon_t \) is multivariate-Gaussian implies that the conditional probability density function of \( X_t \) is

\[
f (X_t \mid y_{o,t-1}, R_{t-1}) = \frac{\exp\left(-\frac{1}{2}\varepsilon_t^T (\Sigma'\Sigma)^{-1} \varepsilon_t\right)}{\sqrt{(2\pi)^T | \Sigma'\Sigma |}},
\]

(3.9)

Let the \( y_{u,t} \) be a vector of remaining \( N - 3 \) yields priced by the model with an error:

\[
y_{u,t} = A_u + B_u X_t + \xi_t.
\]

The second part of the likelihood function refers to the pricing errors \( \xi_t \). They are assumed to be distributed as i.i.d. \( N(0, \omega^2 I) \), with the identical \( \omega^2 \) and zero correlations across yields. \( I \) is a unity matrix. The conditional density of \( y_{u,t} \) is thus given by
\[ f(y_{u,t} \mid X_t) = \frac{\exp \left( -\frac{1}{2} \xi_t' \left( (\omega^2 I)' \omega^2 I \right)^{-1} \xi_t \right)}{\sqrt{(2\pi)^T |(\omega^2 I)' \omega^2 I|}}. \]  

(3.10)

The log-likelihood function is just the sum of logarithms of the “time-series part” in equation (3.9) and the “cross-sectional part” in equation (3.10) and thus takes the form

\[ \ln \mathcal{L}(\cdot) = \log(f(X_t \mid y_{o,t-1}, \text{Reserves}_{t-1})) + \log(f(y_{u,t} \mid X_t)). \]  

(3.11)

5.2 Econometric Identification

Solid identification of parameters is an essential part of dynamic term structure model estimation. The proposed identification scheme follows Dai and Singleton (2000) and Hamilton and Wu (2010a).

To begin with, the upper-left block of the VAR matrix \( \Psi \), i.e. the matrix driving the dynamics of the yields-only factors in \( X_{o,t} \), is set to be a power law structure,\(^{23} \) with zero non-diagonal elements and the following power relation on the diagonal:

\[ \psi_{zz} = \psi_{11} \alpha^{z-1}. \]

\( \psi_{11} \) is the largest eigenvalue of \( \Psi \). The AR coefficient of the first latent factor \( \alpha \) serves as a scaling parameter controlling the distance between the eigenvalues, and \( z = 2, \ldots, Z \), where \( Z \) is the number of latent factors. Preliminary estimation showed that the \( \psi_{11} \) parameter is near one. In line with the near co-integration assumption from previous studies,\(^{24} \) we simply set \( \psi_{11} \) to 1,\(^{25} \) and estimate \( \psi_{zz} \), where \( z = \{2, 3\} \), together with the AR(1) coefficient of the reserves dynamics. The off-diagonal elements of the \( \Psi \) are set to zero, as well as the \( \mu \) vector and the off-diagonal elements of matrix \( \Sigma \) in the transition equation (3.5).

We impose the usual boundary condition \( A_0 = B_0 = 0 \) on the parameters of the pricing

\(^{23}\)See also Calvet et al. (2010) and Bauer and de los Rios (2011).

\(^{24}\)See for instance Giese (2008) and Jardet et al. (2011).

\(^{25}\)As in Diebold and Li (2006), Söderlind (2010) and Bauer (2011).
equation given in (3.7). $A_1$ is normalized to average the one-period interest rate in the sample while $B_1$ is normalized to $[1 \ 1 \ 1 \ b_{Reserves}]'$. Finally, the market price of risk dynamics is restricted to

$$
\lambda_t^s = \begin{bmatrix}
  \lambda_{0,1} \\
  \lambda_{0,2} \\
  0 \\
  0
\end{bmatrix} + \begin{bmatrix}
  0 & 0 & 0 & \lambda_{1,14} \\
  0 & 0 & 0 & \lambda_{1,24} \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0
\end{bmatrix} X_t, \quad (3.12)
$$

so that both “level” and “slope” risks are priced in yields. This is the main market price of risk specification of the study. The results are also reported for the risk-neutral $Q$ measure, where $\lambda_0 = \lambda_1 = 0$ and for the case where only the “level” shock is being a compensated risk, i.e. where $\lambda_{0,2}$ and $\lambda_{1,24}$ are set to 0.

### 5.3 Bayesian Inference

The yield curve implied by the model is a complicated non-linear function of the underlying parameters. As such non-linearity tends to produce a multi-modal likelihood function, it is a daunting task to fit a yield curve model with a standard maximum likelihood estimation. The Bayesian Markov Chain Monte Carlo (MCMC) method is a powerful alternative that provides both efficiency and tractability.

**Setting**

Let $\Theta$ be a vector of length $K$ that collects all the parameters of the model to be estimated:

$$
\Theta = \{\alpha, \psi_R, \Sigma, \lambda_0, \lambda_1, \omega, b_R\}.
$$

---

26 Following Favero et al. (2007).

27 As in Ang et al. (2008). Alternatively, one could set the covariance matrix of the transition equation (3.5) to a unity matrix and estimate all the elements of $B_1$ as for instance in Ang and Piazzesi (2003).

28 As in Duffee (2010) and Joslin et al. (2010).

29 We follow the specification in Cochrane and Piazzesi (2009) and add the slope factor. It is indeed a restricted set of models, yet previous studies show that many restrictions on the market price of risk are supported by the data, see Joslin et al. (2010) and Bauer (2011).

The key idea behind Bayesian estimation is to consider the vector as a multivariate random variable and to use the Bayes’ rule to “learn” about the variable given the data observed. The posterior density of $\Theta$

$$p(\Theta \mid \text{data}) = \frac{p(\text{data} \mid \Theta)p(\Theta)}{p(\text{data})},$$  

(3.13)

where $p(\text{data} \mid \Theta)$ is the likelihood function and $p(\Theta)$ denotes the prior density of the parameters. The term $p(\text{data})$ is known as "normalizing constant" and is independent of $\Theta$.\(^{31}\) Consequently, the rule in (3.13) can be re-written as

$$\ln p(\Theta \mid \text{data}) \propto \ln L + \ln p(\Theta),$$

where

$$\ln L = \sum_{t=0}^{T-1} \ln \text{pdf}(X_t \mid y_{o,t-1}, r_{t-1})$$

$$+ \sum_{t=0}^{T-1} \ln \text{pdf}(y_{u,t} \mid X_t, y_{o,t-1}, r_{t-1})$$

(3.14)

is the logarithm of the likelihood function defined in (3.9) and (3.10).

**Priors**

In the estimation, the priors $p(\Theta)$ are set to be non-informative or "flat", so that the posterior density of the model parameters is drawn with equal probability from the pre-defined support interval. Alternatively, we could derive the prior distributions for parameters $\Psi$ and $\Sigma$, given the normality assumption of the state VAR process,\(^{32}\) and for $\omega$ given the assumption of the Gaussian measurement error.\(^{33}\) Chib (2001) proposes a scaled beta distribution as an alternative to the uniform distribution. Nevertheless, we choose not to

\(^{31}\)In particular, $p(\text{data}) = \int p(\text{data} \mid \Theta)p(\Theta)d\Theta$. See Koop (2003).

\(^{32}\)For instance, see Ang et al. (2007).

\(^{33}\)See for example Mikkelsen (2001).
impose lower (or higher) probability areas from which the candidate values of parameters are drawn. This way, the estimation is almost completely data-driven and proves to be computationally efficient.

The parameters’ support intervals are specified by following the no-arbitrage condition and previous studies. In particular, the eigenvalues of the VAR matrix are set to be positive and less than one and the volatility parameters on the diagonal of $\Sigma$ are set to be non-negative. The parameter $b_{NBR}$ is constrained to lie inside the unit-circle. The lower bound of the parameters in vector $\lambda_0$ and matrix $\lambda_1$ are set as in Chib and Ergashev (2009).

**Markov Chain Monte Carlo**

We use a simple version of the “Metropolis within Gibbs” algorithm\(^{34}\) to draw the parameters from their posterior densities. The parameter candidates are drawn from continuous uniform distributions $U(\Theta, \overline{\Theta})$ where the lower and the upper boundaries $\Theta$ and $\overline{\Theta}$ for each parameter in $\Theta$ are specified in Table 3.2. The algorithm can be described in five steps.

Step 1: Set the initial values for the parameters $\Theta^0$. Set up two Markov chains with different starting values. The initial values for the VAR parameters in the first chain are obtained from OLS and the data descriptive statistics. The starting values of the market price of risk parameters and of the second chain are chosen arbitrarily.\(^{35}\)

Step 2: Draw a candidate log-posterior density $\ln p(\Theta^* | \Theta^{mc-1}, \text{data})$ conditional on previously drawn parameter values $\Theta^{mc-1}$. The number $mc$ denotes current iteration. The

\(^{34}\)The “Metropolis within Gibbs” algorithm is a simple method and therefore often used in the literature, as for example in Gilks (1996), Koop (2003) and Lynch (2007). In particular, this algorithm features two favorable characteristics. First, in standard Metropolis new proposals for the parameter values are drawn all at once whereas the Gibbs sampler draws only one proposal for only one parameter value at a time. Therefore, the Gibbs sampler results much more efficient, i.e. the estimator converges quicker. Second, the tuning of the estimator is simpler, where tuning refers to setting the scale factor in step 2 to different values. In standard Metropolis, one scale factor must “fit all”. The Gibbs sampler is more flexible as it allows for a separate set of values for each parameter.

\(^{35}\)For example, the volatility parameters’ starting values are set to be 3 times larger in the second chain, the $\Psi$ matrix parameters are set to 0.8 and the $B_1$ parameter is set to 0.5 and -0.5 in the first and the second Markov chain, respectively.
draws are performed separately for every parameter in $\Theta$. For instance, a proposal for the first element in the vector $\Theta$ is generated by the Markov chain

$$\theta^*_1 = \theta^{mc-1}_1 + \nu_1 U_1,$$

where $\nu_k$ is a scaling factor and $U_k$ is an uniformly distributed random number from the interval $[-1,1]$. We initialize $\nu_k$ for parameters $\alpha$ and $\psi_{NBR}$ to 0.01, for the diagonal elements of matrix $\Sigma$ and the market price of risk parameters to 0.1, and for the $\omega$ parameter to 0.00001.\(^{36}\) The scaling factor is then automatically updated after every 5,000 sweeps to obtain an acceptance ratio in step 4 of approximately 0.5.

Step 3: For every parameter in $\Theta^*$, calculate the difference between the posterior density of the candidate value and the posterior density of the previously drawn parameter value, keeping the other parameter values unchanged. Using again the first element in $\Theta$ as an example, the difference reads:

$$\delta = \ln p(\{\theta^*_1,\theta^{mc-1}_2,...,\theta^{mc-1}_K\} | \Theta^{mc-1}, \text{data}) - \ln p(\{\theta^{mc-1}_1,\theta^{mc-1}_2,...,\theta^{mc-1}_K\} | \Theta^{mc-1}, \text{data})$$

(3.15)

Step 4: Draw a random number $u \sim U(0,1)$ and accept the single parameter candidate from Step 2 whenever

$$\min(0, \delta) > \log(u)$$

holds for the difference in Step 3.

Step 5: Repeat steps 2 to 5 until the joint posterior density of parameters converges in distribution.

\(^{36}\)Proposed in Ang et al. (2007) so that it roughly corresponds to a 30 basis points bid-ask spread on Treasuries. An average spread on the OTC plain vanilla swap market might be similar. See also Skarr (2010).
The algorithm is run 100,000 times and the first 40,000 are discarded as the burn-in period. The two Markov chains with different starting values for both joint posterior and the single parameters’ posteriors converge to literally the same posterior distributions. Before estimating the entire model, the proposed parametrization is used to estimate the risk neutral specification. The model under $Q$ converges even quicker and thus the algorithm is run for 50,000 times and the first 20,000 are discarded as burn-in.

## 6 Results

QE may affect either the expectations’ part of interest rates or the term premium - or both. Term structure models as applied in this paper are suitable to isolate the term premium. Therefore, the three latent factors should account for changes in expected future interest rates. Central bank reserves as a fourth factor are added to explain changes in the term premium. Our model performs well in that most of the variation in yields is explained by three latent factors. The fourth factor contributes only marginally and it is only slightly correlated with the latent factors. Our estimates show that QE has significantly lowered the term premium of long-term Treasury yields. Estimates of the cumulative effect of QE on 10-year Treasury yields during the ZLB amounts to 85 basis points. This result is in line with the recent literature. Estimates of Swiss data suggest 10-year yields decreased 34 basis points which can be fully ascribed to the liquidity effect because the SNB did not purchase government bonds to expand liquidity. In the following, this section lays out the model performance and then presents the estimation results and the economic interpretation. Finally, the supply and the liquidity effect are disentangled. As described above, the model is estimated under three different settings concerning the price of risk. Accordingly, the results are reported for the three different models.

### 6.1 Model Performance

Table 3.2 and ?? report the parameter estimates for the two subsamples for US and Swiss data, respectively. The tables report the estimated posterior modes of the parameters.

---

37 On average after 7,000 sweeps, between the non-ZLB sample and the ZLB sample.

38 The scaling factor is also automatically adapted until the 20,000th iteration.
together with numerical standard errors (in brackets) for the subsample away from the ZLB ($\Theta_{\text{nonZLB}}$) and the subsample at the ZLB ($\Theta_{\text{ZLB}}$). The first two columns provide the support intervals, and the last two the average acceptance ratios and inefficiency factors (IF) for the two subsamples.\footnote{The two indicators are explained in the tables’ headings.}

Table 3.4 reports the cross-sectional fit for both countries and the two subsamples. By construction, the 3-month, the 5-year and the 10-year yields are explained perfectly by the model. The results are therefore not reported for these maturities. During the ZLB period, the shorter end of the yield curve approached zero. This lower bound brought down interest rate volatility at shorter maturities. Therefore, the pricing performance during the ZLB period clearly outperforms the pre-ZLB pricing performance at the shorter end of the yield curve. Comparing the different market price of risk settings, the time-varying level and slope risks clearly improve the fit above the $\mathcal{Q}$ model. This is true especially in the second subsample at mid to long maturities. Letting NBR affect the market price of risk considerably improves the fit only in the ZLB-subsample. Thus, the choice of market price of risk specification turns out to be relatively more important in the ZLB period than in the pre-crisis sample. The fit also improves in the earlier subsample, but the gain is relatively smaller. Allowing NBR to additionally affect the slope risk improves the fit further at longer maturities only during the ZLB period. During normal times, as in the earlier subsample, the addition of the time-varying slope risk lowers the fit.

A broad consensus in the term structure models literature is that the first three yields-only factors account for most of the movements in the cross-section of yields.\footnote{One of the key contributions in this field is Litterman and Scheinkman (1991).} Figure 3.4 shows the estimated latent factors in the two subsamples. Comparing the evolution of the latent factors with the yield curve depicted in Figure 3.1 shows that the first latent factor seems to drive the level movements of the yield curve in both periods, being relatively stable in the first period and substantially increasing in volatility in the second. The second latent factor $l_2$ seems to move the slope of the yield curve, but this is only clear in the non-ZLB period. The third latent factor $l_3$ co-moves to a great extent with the second factor $l_2$.\footnote{The two indicators are explained in the tables’ headings.}
The correlation in levels between the two factors amounts to -0.75 and -0.94 in the first and the second subsample, respectively. Ang and Piazzesi (2003) and Joslin et al. (2011) show that the entire yield curve can be explained fairly well with three latent factors, and that adding macro variables does not improve a model. The variance decomposition presented in Table 3.5 satisfies the conjecture.\(^{41}\) It indicates the fraction of the total variance explained by each factor. The fourth factor, i.e. reserves, adds only marginal information to the model while most of the cross-variation in yields is explained by the latent factors.\(^{42}\) Most of the cross-sectional variation in yields is given by the first two latent factors. In the earlier subsample, the first two latent factors explain 59% of the variance of yields at the 6-month horizon. This fraction increases with maturity and amounts to over 96% at the 9-year horizon. The model performs similarly well in the ZLB subsample: the fraction of variance explained by the first two factors increases from 70% at the shortest to 94% at the longest maturity. With the third latent factor in addition, practically the entire variance is explained. The contribution of NBR does not exceed 0.1% at any of the maturities in the first subsample. At the ZLB, however, this contribution of the NBR factor increases considerably. Liquidity explains around 1% of total variation at the shorter end of the yield curve and increases to almost 4% at longer maturities. In the Swiss case, the marginal contribution of giro holdings falls at short maturities but quadruples at long maturities when entering the ZLB period. Hence, at the ZLB, the size of giro holdings has gained importance in explaining long-term yields.

The dynamics of the NBR factor is found to be independent from the yields-only factors. Table 3.1 reports the correlation of NBR with the three factors: in the first subsample, the correlations are -0.09, 0.19, and -0.22, respectively. In the second subsample, the correlations decrease in size to 0.07, 0.11, and 0.07. This shows that the NBR factor adds information to the analysis that is complementary to the information contained in the yield curve.

\(^{41}\)Table 3.5 reports the variance decomposition for selected yields. An instructive derivation thereof can be found in Dahlquist and Hasseltoft (2011).

\(^{42}\)Our preliminary analysis also showed that the reserves factor contribution to pricing performance was minimal. We do not report this result.
6.2 Parameters

The coefficient estimates are depicted in the third and fourth column of Table 3.2 and ?? for the US and Switzerland, respectively. To start at the bottom of Table 3.2, the estimated loading of the NBR factor on the short rate is significantly negative in both subsamples. This is in line with the hypothesis that expansionary monetary policy has a negative effect on yields. In the ZLB sample, the coefficient is much larger in size which suggests that a one standard deviation increase has had a larger impact on yields at the ZLB bound than in normal times. Opposed to the US estimates, the bottom of Table ?? shows that Swiss data does not provide a significant coefficient on the loading of banks giro holdings.

$\lambda_{01}$ and $\lambda_{02}$ refer to the time-invariant market price of risk associated to the level and the slope of the yield curve, respectively. In both countries, the market price of level risk fell substantially when the ZLB was hit.43 This reflects the fact that zero is a natural bound for interest rates where the risk becomes one sided. Conversely, the slope risk increased. There are many possible reasons for elevated uncertainty with respect to future rates: it may reflect uncertainty about the length of the financial crisis or it could reflect uncertainty not only about the future path of unconventional monetary policy but also about its consequences.

The time-varying prices of risk coefficients, $\lambda_{14}$ and $\lambda_{24}$, do not significantly differ from zero in the pre-crisis subsample in either of the two countries. At the ZLB, however, they become statistically significant. $\lambda_{14}$ is associated with the effect of NBR (or giro holdings) on the market price of risk in the interest rate level. The significantly negative $\lambda_{14}$ in both countries suggests that during the ZLB period, the discount factor decreases with an expansion in NBR, and so the interest rate risk increases in the level. This could be evidence that QE elevated uncertainty with respect to future interest rates or inflation which in turn affects the term premium. For example, expansionary monetary policy will eventually lead to higher inflation rates which, anticipated in the form of increasing inflation expectations, drive up nominal interest rates (Fisher effect). The term premium increases with uncer-

43Note that $\lambda$ refers to the discount factor of a bond price. A higher coefficient thus means a lower market price of risk in the yield.
tainty associated with inflation expectations. Evidence of mechanisms such as the supply or the liquidity effect is found in the market price of risk associated with the slope of the yield curve. $\lambda_{24}$ turns significantly positive at the ZLB which suggests that an increase in NBR would lead to a flattening of the yield curve. Hence, with short-term interest rates already close to zero, the supply and the liquidity effects affect the longer end of the yield curve.

6.3 The Estimated Effect of Reserves on Interest Rates

The estimation results of the factor loadings are reported in Table 3.6 and 3.7 for the US and Switzerland, respectively. The upper tables refer to the pre-crisis subsample, the lower panels to the ZLB subsample. The factor loadings of reserves on yields are presented for selected maturities and the three different market price of risk settings as described above. The factor loadings indicate by how much yields change in response to a one standard deviation change in NBR. One standard deviation of NBR corresponds to $2.3$ billion in the pre-ZLB period and rises up to $315.8$ billion in the ZLB subsample. For the Swiss giro holdings, one standard deviation amounts to CHF 0.633 billion in the early sample and to CHF 43.524 billion during the ZLB period.

Estimation Results

There is only weak evidence for an effect of reserves on yields in the pre-crisis subsample. The traditional view of the liquidity effect would call for a negative effect of an increase in liquidity on the short end of the yield curve. As the upper panel of Table 3.6 shows, a one standard deviation increase in NBR is followed by a decrease of the 3-month yield of 1 to 2 basis points, depending on the model. This estimate is not significantly different from zero according to the Z-test, however. As expected, there is no effect of reserves on longer-term yields in the early sample in either of the two countries. This result is robust with respect to the different market price of risk settings. The remainder of this section

---

44For the Z-test, the null hypothesis is a zero-mean and the standard error is calculated from the estimation output.
therefore focusses on the ZLB period.

During the ZLB period, the effect of reserves on yields becomes significant in both countries at all maturities. As shown in the lower panel of Table 3.6, NBR have affected the yield curve differently across the models. The third model - which allows for time variation in the prices of both level and slope risk - indicates a particularly pronounced effect at mid to longer maturities. The other two models suggest a significant impact at all maturities. The first model - which allows for no time variation in the prices of risk - estimates a constant effect across maturities. The second model - which allows only the price of level risk to vary over time - exhibits an effect that is decreasing with maturity. The estimates for Switzerland give a very similar picture: During the ZLB period, giro holdings significantly affected the yield curve, including at the longer end. The relative effect across maturities in unclear as it varies depending on the model settings. In Swiss data, the effect seems to be fairly stable across maturities.

Size of the Effect Relative to GDP

At the 10-year horizon, a one standard deviation increase in NBR has lowered yields between 4 to 26 basis points according to our estimates. This amounts to 1 to 8 basis points per increase in NBR of USD 100 billion. With a nominal GDP of roughly $15.1 trillion, this translates into an increase of approximately 2 to 12 basis point due to an increase in NBR of the size of one percent of GDP. Accordingly, the Swiss data yields an effect of 3 to 7 basis points per standard deviation change in liquidity, or 7 to 16 basis points per CHF 100 billion. With a Swiss GDP of approximately CHF 565 billion, an increase in liquidity in the range of one percent of nominal GDP lowers rates by 0.4 to 0.9 basis points. Hence, the total effect of the liquidity expansion on long-term yields is estimated to be considerably larger in the US compared to Switzerland.

45 The average USDCHF exchange rate in the period of the Swiss ZLB subsample from mid December to the end of September 2011 was 1.0278, so CHF 100 billion is comparable to USD 97 billion.
Cumulative Effect During the ZLB

Figure 3.5 plots the estimated cumulative impact of an increase in NBR on Treasury yields during the ZLB period, Figure 3.6 depicts the equivalent for Switzerland. The graphs on the left refer to the 3-month yield, the graphs on the right to the 10-year rate. The upper row shows the estimates from the model under the $Q$ measure, the middle row to the model allowing the level risk of interest rates to vary over time, and the bottom row to the model allowing both the level and slope risks of interest rates to be time varying. As the 90% confidence bands indicate the statistically significant effect of reserves on yields. The total increase in NBR of roughly 1,028 billion USD is associated with a fall in the 5-year yield of 98 basis points and a fall in the 10-year yield of 85 basis points.

The other two market price of risk specifications estimate a smaller effect. The model under the $Q$-measure estimates a total decrease of the 10-year Treasury yields of roughly 13 basis points due to the increase in NBR. The model accounting for a time-varying level risk estimates a fall in the long-term yields of approximately 52 basis points.

In Switzerland the cumulative effect of the expansion in giro holdings from approximately 26 billion CHF in December 2008 to 235 billion CHF in October 2011 has led to an estimated fall in long-term yields between 14 to 34 basis points. The 90%-confidence bands are rather large as Figure 3.6 shows, however. But in all models, the cumulative impact is significantly different from zero.

The size of the estimated impact for US data is in line with the existing literature. The 26 basis point fall in rates per standard deviation in NBR corresponds to an effect of 8 basis points per USD 100 billion. Gagnon et al. (2011) find a total supply effect 30 to 100 basis points which translates into 2 to 6 basis points per 100 billion USD. Hamilton and Wu (2010a) estimate a range of 17 to 48 basis points, equivalent to 12 to 17 basis points per USD 100 billion. D’Amico and King (2010) estimate an effect of 50 basis points in response to purchases of USD 300 billion (17 basis points per USD 100 billion), and Krishnamurthy and Vissing-Jorgensen (2011) estimate the total effect of the LSAP purchases to amount to roughly 100 basis points. Finally, Krogstrup et al. (2012) estimate an effect of 26
basis points per USD 100 billion of outright Treasury purchases on 10-year Treasury yields.

6.4 Disentangling the Different Effects

The signalling effects of monetary policy as well as expectations about the future path of interest rates should be captured by the first three factors of the model. Hence, the fourth factor contributes information on the term premium. This paper compares the estimates of US data with those of Swiss data to separate the supply effect from the liquidity effect. The former cannot be present in Swiss data because the SNB has not bought Swiss government bonds during the ZLB period. Up to our knowledge, the only paper that tries to disentangle the two effects is Krogstrup et al. (2012) who conduct the study using US data. According to their estimates, the cumulative liquidity effect exceeds the supply effect by a factor of three.

The fact that there is a significant effect of giro holdings on Swiss long-term yields provides evidence for the liquidity effect. Per standard deviation change in reserves, the effect in the US results up to almost four times the effect in Switzerland. In terms of a 1% increase of GDP, the estimated effect for the US exceeds the one for Switzerland by up to the twelvefold. This suggests that in the US, there were other effects at play on top of the liquidity effect that were not at play in Switzerland. We conclude from these results that the liquidity effect is between a twelfth and a quarter the size of the supply effect. If three quarters of the estimated impact in the US are due to supply effects and one quarter to liquidity effects, the former would amount to and accumulated effect of 64 basis points and the latter to 21 basis points during the ZLB sample.

Figure 3.7 shows the development of NBR and the Federal Reserves holdings of Treasury securities with maturities of more than one year. It shows that the correlation is limited. The Treasury securities increased in two steps: in Q2 of 2009 and again in Q4 2011. NBR increased the most in Q3 of 2009 and fluctuated much more than the Treasury holdings. Assuming that our estimates capture the liquidity effect and the supply effect jointly, the difference in the change of NBR and the change in Treasury holdings would be a good
measure of the “pure” liquidity increase. The lower panel of Figure 3.7 draws this “pure” cumulative liquidity effect. The bold black line depicts the liquidity effect with the smallest factor loading of 4 basis points per standard deviation which results from the model under the Q-measure. The dashed red line depicts the liquidity effect with the largest factor loading of 26 basis points per standard deviation estimated by the third model. These computations suggest that the cumulative “pure” liquidity effect amounted to 3 to 20 basis points. The rest of the effect is attributable to a mix between the supply and the liquidity effect. Thus, the 20 basis points most likely underestimate the cumulative liquidity effect.

Finally, information theory would suggest that supply effects are to a great extent priced in at the moment of the announcement (or even earlier if expected). If this is true, then NBR can only explain supply effects of Treasury securities that exceeded expectations. Conversely, the liquidity effect can only work once reserves have increased. Thus, our estimates for the liquidity effect are likely to underestimate the true liquidity effect of QE on the yield curve.

7 Conclusion

QE may affect either the expectations’ part of interest rates or the term premium - or both. Term structure models as applied in this paper are suitable to isolate the term premium. Therefore, three latent factors should account for changes in expected future interest rates. Central bank reserves as a fourth factor are added to explain changes in the term premium. Our estimates show that QE has significantly lowered the term premium of long-term Treasury yields. The results suggest that an increase in liquidity has affected the entire yield curve at the ZLB in both US and Swiss data. The impact amounts up to 8 basis points per USD 100 billion in the US. This is line with the literature that estimates an effect of 2 to 26 basis points per USD 100 billion. On a cumulative basis, our estimates suggest that long-term yields have been lowered by QE measures by up to 85 basis points in the US and 34 basis points in Switzerland.

The Swiss results provide evidence of liquidity effect at the longer end of the yield curve when the short-term interest rates are bounded by the ZLB because the SNB expanded
liquidity by foreign exchange rather than government bond purchases. Therefore, a supply effect can be excluded. While the estimate is entirely attributable to the liquidity effect in Switzerland, the total effect in the US must be divided into the supply effect and the liquidity effect. According to our estimates, the total effect in the US exceeds the one in Switzerland by the threefold. If we deduce from this ratio that one quarter of the total effect in the US is due to the liquidity effect, it would amount to a cumulative 21 basis points in the ZLB sample. The second approach to isolate the liquidity effect yields the same result. We deduct the Treasury supply purchases from the changes in NBR and compute the liquidity effect by solely considering the increases in NBR that were not due to Treasury securities purchases. This yields a “pure” liquidity effect of 20 basis points. The remaining 64 basis points can be ascribed to a mix between the supply and the liquidity effect.
### Table 3.1: Sample Correlations

The table reports contemporaneous correlations of selected interest rates and central bank reserves in levels (upper panel) and in first differences (lower panel) for “normal times” and the ZLB subsamples.

<table>
<thead>
<tr>
<th></th>
<th>Levels</th>
<th>Non-ZLB</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
</tr>
<tr>
<td>USA</td>
<td>-0.39</td>
<td>-0.37</td>
<td>-0.31</td>
<td>-0.26</td>
<td>-0.25</td>
<td>-0.24</td>
</tr>
<tr>
<td>CH</td>
<td>0.19</td>
<td>0.19</td>
<td>0.14</td>
<td>0.16</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>ZLB</td>
<td></td>
<td>3M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
</tr>
<tr>
<td>USA</td>
<td>-0.54</td>
<td>-0.84</td>
<td>-0.61</td>
<td>-0.41</td>
<td>-0.38</td>
<td>-0.36</td>
</tr>
<tr>
<td>CH</td>
<td>-0.28</td>
<td>-0.27</td>
<td>-0.64</td>
<td>-0.69</td>
<td>-0.59</td>
<td>-0.53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>First Differences</th>
<th>Non-ZLB</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>10Y</td>
</tr>
<tr>
<td>USA</td>
<td>0.05</td>
<td>-0.05</td>
<td>-0.21</td>
<td>-0.20</td>
<td>-0.19</td>
<td>-0.19</td>
</tr>
<tr>
<td>CH</td>
<td>0.29</td>
<td>0.10</td>
<td>0.03</td>
<td>-0.02</td>
<td>-0.06</td>
<td>-0.08</td>
</tr>
<tr>
<td>ZLB</td>
<td></td>
<td>3M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
</tr>
<tr>
<td>USA</td>
<td>-0.13</td>
<td>-0.17</td>
<td>-0.17</td>
<td>-0.15</td>
<td>-0.15</td>
<td>-0.15</td>
</tr>
<tr>
<td>CH</td>
<td>-0.40</td>
<td>-0.44</td>
<td>-0.28</td>
<td>-0.12</td>
<td>-0.10</td>
<td>-0.12</td>
</tr>
</tbody>
</table>
Table 3.2: Parameter Estimates for the US. The table reports the estimated posterior modes of the parameters for the US, together with numerical standard errors (in brackets) for the subsample away from the ZLB ($\Theta_{\text{nonZLB}}$) and the subsample at the ZLB ($\Theta_{\text{ZLB}}$). The first two columns provide the support intervals, and the last two the average acceptance ratios and inefficiency factors (IF) for the two subsamples. The acceptance ratio is the number of accepted parameters’ proposals divided by the number of iterations after burn-in. The rate between 0.25 and 0.75 is often acceptable, see Lynch (2007) and Koop (2003). The inefficiency factor (IF) of how well the sampler “mixes” is computed as $1 + 2\sum_{l=1}^{L} \rho(l)$, where $\rho(l)$ is the autocorrelation at lag $l$ in the Markov chain sequence of a parameter, and $L$ is the lag at which the autocorrelation function goes to zero. See Chib (2001) for details.

<table>
<thead>
<tr>
<th></th>
<th>$\Theta$</th>
<th>$\Theta_{\text{nonZLB}}$</th>
<th>$\Theta_{\text{ZLB}}$</th>
<th>AccRatio</th>
<th>IF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.00</td>
<td>0.99</td>
<td>0.92</td>
<td>0.93</td>
<td>0.70</td>
</tr>
<tr>
<td>$\psi_{\text{NBR}}$</td>
<td>0.00</td>
<td>1.00</td>
<td>0.84</td>
<td>1.00</td>
<td>0.52</td>
</tr>
<tr>
<td>$\sigma_{11}$</td>
<td>0.00</td>
<td>15.0</td>
<td>0.08</td>
<td>0.14</td>
<td>0.20</td>
</tr>
<tr>
<td>$\sigma_{22}$</td>
<td>0.00</td>
<td>15.0</td>
<td>0.22</td>
<td>0.33</td>
<td>0.41</td>
</tr>
<tr>
<td>$\sigma_{33}$</td>
<td>0.00</td>
<td>15.0</td>
<td>0.30</td>
<td>0.26</td>
<td>0.20</td>
</tr>
<tr>
<td>$\sigma_{\text{NBR}}$</td>
<td>0.00</td>
<td>15.0</td>
<td>1.07</td>
<td>0.09</td>
<td>0.33</td>
</tr>
<tr>
<td>$\lambda_{01}$</td>
<td>-100</td>
<td>100</td>
<td>1.87</td>
<td>5.10</td>
<td>0.57</td>
</tr>
<tr>
<td>$\lambda_{02}$</td>
<td>-100</td>
<td>100</td>
<td>0.83</td>
<td>-2.00</td>
<td>0.26</td>
</tr>
<tr>
<td>$\lambda_{14}$</td>
<td>-100</td>
<td>100</td>
<td>-0.01</td>
<td>-0.08</td>
<td>0.80</td>
</tr>
<tr>
<td>$\lambda_{24}$</td>
<td>-100</td>
<td>100</td>
<td>0.00</td>
<td>0.09</td>
<td>0.68</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.00</td>
<td>10.0</td>
<td>0.21</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>$b_{\text{NBR}}$</td>
<td>-1.00</td>
<td>1.00</td>
<td>-0.02</td>
<td>-0.24</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Table 3.3: *Parameter Estimates for Switzerland.* The table reports the estimated posterior modes of the parameters for Switzerland, together with numerical standard errors (in brackets) for the subsample away from the ZLB ($\Theta_{\text{nonZLB}}$) and the subsample at the ZLB ($\Theta_{\text{ZLB}}$). The first two columns provide the support intervals, and the last two the average acceptance ratios and inefficiency factors (IF) for the two subsamples. The acceptance ratio is the number of accepted parameters’ proposals divided by the number of iterations after burn-in. The inefficiency factor (IF) of how well the sampler “mixes” is computed as $1 + 2 \sum_{l=1}^{L} \rho(l)$, where $\rho(l)$ is the autocorrelation at lag $l$ in the Markov chain sequence of a parameter, and $L$ is the lag at which the autocorrelation function goes to zero. See Chib (2001) for details.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Theta$</th>
<th>$\Theta_{\text{nonZLB}}$</th>
<th>$\Theta_{\text{ZLB}}$</th>
<th>AccRatio</th>
<th>IF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.00</td>
<td>0.99</td>
<td>0.90</td>
<td>0.92</td>
<td>0.66</td>
</tr>
<tr>
<td>$\psi_{\text{SightDepo}}$</td>
<td>0.00</td>
<td>1.00</td>
<td>0.99</td>
<td>1.00</td>
<td>0.62</td>
</tr>
<tr>
<td>$\sigma_{11}$</td>
<td>0.00</td>
<td>15.0</td>
<td>0.06</td>
<td>0.08</td>
<td>0.22</td>
</tr>
<tr>
<td>$\sigma_{22}$</td>
<td>0.00</td>
<td>15.0</td>
<td>0.21</td>
<td>0.18</td>
<td>0.38</td>
</tr>
<tr>
<td>$\sigma_{33}$</td>
<td>0.00</td>
<td>15.0</td>
<td>0.27</td>
<td>0.17</td>
<td>0.20</td>
</tr>
<tr>
<td>$\sigma_{\text{SightDepo}}$</td>
<td>0.00</td>
<td>15.0</td>
<td>0.83</td>
<td>0.17</td>
<td>0.25</td>
</tr>
<tr>
<td>$\lambda_{01}$</td>
<td>-100</td>
<td>100</td>
<td>1.58</td>
<td>1.95</td>
<td>0.60</td>
</tr>
<tr>
<td>$\lambda_{02}$</td>
<td>-100</td>
<td>100</td>
<td>0.31</td>
<td>-1.06</td>
<td>0.27</td>
</tr>
<tr>
<td>$\lambda_{14}$</td>
<td>-100</td>
<td>100</td>
<td>0.01</td>
<td>-0.04</td>
<td>0.81</td>
</tr>
<tr>
<td>$\lambda_{24}$</td>
<td>-100</td>
<td>100</td>
<td>-0.01</td>
<td>0.10</td>
<td>0.75</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.00</td>
<td>10.0</td>
<td>0.20</td>
<td>0.20</td>
<td>0.21</td>
</tr>
<tr>
<td>$b_{\text{SightDepo}}$</td>
<td>-1.00</td>
<td>1.00</td>
<td>-0.01</td>
<td>0.03</td>
<td>0.23</td>
</tr>
</tbody>
</table>

106
Table 3.4: Pricing Errors. Mean absolute pricing errors in basis points across different modelling assumptions are reported for selected yields in the US (upper panel) and Switzerland (lower panel), and for the two subsamples.

<table>
<thead>
<tr>
<th></th>
<th>USA</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>non-ZLB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>9Y</td>
</tr>
<tr>
<td>Q</td>
<td>11.6</td>
<td>12.5</td>
<td>3.5</td>
<td>1.9</td>
<td>4.3</td>
<td>3.4</td>
</tr>
<tr>
<td>l</td>
<td>9.7</td>
<td>7.9</td>
<td>3.1</td>
<td>1.1</td>
<td>1.7</td>
<td>1.3</td>
</tr>
<tr>
<td>l &amp; s</td>
<td>10.1</td>
<td>8.9</td>
<td>3.3</td>
<td>1.3</td>
<td>2.3</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>ZLB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>9Y</td>
</tr>
<tr>
<td>Q</td>
<td>5.5</td>
<td>8.0</td>
<td>6.6</td>
<td>4.2</td>
<td>5.2</td>
<td>3.0</td>
</tr>
<tr>
<td>l</td>
<td>3.6</td>
<td>6.6</td>
<td>3.1</td>
<td>1.1</td>
<td>2.0</td>
<td>1.4</td>
</tr>
<tr>
<td>l &amp; s</td>
<td>4.2</td>
<td>7.5</td>
<td>3.6</td>
<td>1.1</td>
<td>1.6</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>CH</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>non-ZLB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>9Y</td>
</tr>
<tr>
<td>Q</td>
<td>6.7</td>
<td>12.7</td>
<td>5.0</td>
<td>1.1</td>
<td>2.7</td>
<td>4.7</td>
</tr>
<tr>
<td>l</td>
<td>9.2</td>
<td>17.0</td>
<td>3.8</td>
<td>0.9</td>
<td>3.0</td>
<td>4.9</td>
</tr>
<tr>
<td>l &amp; s</td>
<td>9.0</td>
<td>16.6</td>
<td>3.7</td>
<td>0.9</td>
<td>3.0</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>ZLB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>9Y</td>
</tr>
<tr>
<td>Q</td>
<td>14.2</td>
<td>10.0</td>
<td>6.2</td>
<td>1.5</td>
<td>7.6</td>
<td>3.9</td>
</tr>
<tr>
<td>l</td>
<td>14.9</td>
<td>11.0</td>
<td>5.2</td>
<td>1.5</td>
<td>7.9</td>
<td>4.0</td>
</tr>
<tr>
<td>l &amp; s</td>
<td>15.1</td>
<td>11.0</td>
<td>5.3</td>
<td>1.4</td>
<td>7.1</td>
<td>3.4</td>
</tr>
</tbody>
</table>

107
Table 3.5: **Variance Decomposition.** The table reports variance decomposition of selected yields in percent for the US (upper panel) and Switzerland (lower panel). The variance is decomposed by dividing each single state variable shock $j$ to an $n$-periods yield: $MSE_n^j = B_n' \Sigma^j B_n + B_n' \Psi \Sigma^j \Psi B_n$, where $\Sigma^j$ is a $K \times K$ matrix with zeros and a non-zero $jj$ element corresponding to the volatility of state variable $j$; with the overall mean squared error of forecasting the states 1 period ahead: $MSE_n = B_n' \Sigma B_n + B_n' \Psi \Sigma \Psi B_n$.

<table>
<thead>
<tr>
<th></th>
<th>US non-ZLB</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>9Y</td>
</tr>
<tr>
<td>$l_1$</td>
<td>17.4</td>
<td>22.1</td>
<td>40.5</td>
<td>62.3</td>
<td>72.3</td>
<td>76.1</td>
</tr>
<tr>
<td>$l_2$</td>
<td>41.6</td>
<td>44.8</td>
<td>44.6</td>
<td>31.5</td>
<td>23.6</td>
<td>20.5</td>
</tr>
<tr>
<td>$l_3$</td>
<td>40.9</td>
<td>33.0</td>
<td>14.8</td>
<td>6.2</td>
<td>4.1</td>
<td>3.4</td>
</tr>
<tr>
<td>NBR</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>ZLB</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>9Y</td>
</tr>
<tr>
<td>$l_1$</td>
<td>23.2</td>
<td>27.6</td>
<td>43.7</td>
<td>62.0</td>
<td>70.7</td>
<td>74.2</td>
</tr>
<tr>
<td>$l_2$</td>
<td>47.1</td>
<td>48.4</td>
<td>44.1</td>
<td>30.5</td>
<td>23.1</td>
<td>20.1</td>
</tr>
<tr>
<td>$l_3$</td>
<td>28.8</td>
<td>22.8</td>
<td>9.7</td>
<td>3.9</td>
<td>2.5</td>
<td>2.1</td>
</tr>
<tr>
<td>NBR</td>
<td>0.9</td>
<td>1.2</td>
<td>2.5</td>
<td>3.6</td>
<td>3.7</td>
<td>3.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>CH non-ZLB</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>9Y</td>
</tr>
<tr>
<td>$l_1$</td>
<td>34.3</td>
<td>40.8</td>
<td>61.3</td>
<td>79.5</td>
<td>86.2</td>
<td>88.5</td>
</tr>
<tr>
<td>$l_2$</td>
<td>43.4</td>
<td>42.9</td>
<td>32.9</td>
<td>18.5</td>
<td>12.6</td>
<td>10.6</td>
</tr>
<tr>
<td>$l_3$</td>
<td>21.0</td>
<td>15.2</td>
<td>5.1</td>
<td>1.7</td>
<td>1.1</td>
<td>0.9</td>
</tr>
<tr>
<td>Giro</td>
<td>1.3</td>
<td>1.2</td>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>ZLB</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>9Y</td>
</tr>
<tr>
<td>$l_1$</td>
<td>43.8</td>
<td>49.8</td>
<td>67.9</td>
<td>82.9</td>
<td>88.2</td>
<td>90.1</td>
</tr>
<tr>
<td>$l_2$</td>
<td>41.8</td>
<td>40.0</td>
<td>28.9</td>
<td>15.8</td>
<td>10.7</td>
<td>9.0</td>
</tr>
<tr>
<td>$l_3$</td>
<td>14.0</td>
<td>10.0</td>
<td>3.2</td>
<td>1.1</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>Giro</td>
<td>0.3</td>
<td>0.2</td>
<td>0.0</td>
<td>0.3</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Table 3.6: *Factor Loadings for the US.* The table reports the values of the NBR factor loadings for selected yields, together with the Z-score from the Z-test (in parenthesis) and for the non-ZLB period (upper panel) and the ZLB period (lower panel). The levels of significance of 0.1, 0.05 and 0.01 are denoted with *, ** and ***, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Q non-ZLB</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>10Y</td>
</tr>
<tr>
<td>B</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.00</td>
<td>-0.00</td>
<td>-0.00</td>
<td>-0.00</td>
</tr>
<tr>
<td>Z-score</td>
<td>(-0.76)</td>
<td>(-0.77)</td>
<td>(-0.77)</td>
<td>(-0.76)</td>
<td>(-0.76)</td>
<td>(-0.75)</td>
</tr>
<tr>
<td></td>
<td>3M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>10Y</td>
</tr>
<tr>
<td>B</td>
<td>-0.01</td>
<td>-0.00</td>
<td>-0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Z-score</td>
<td>(-0.73)</td>
<td>(-0.61)</td>
<td>(-0.26)</td>
<td>(0.13)</td>
<td>(0.29)</td>
<td>(0.41)</td>
</tr>
<tr>
<td></td>
<td>3M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>10Y</td>
</tr>
<tr>
<td>B</td>
<td>-0.02</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.00</td>
<td>-0.00</td>
<td>-0.00</td>
</tr>
<tr>
<td>Z-score</td>
<td>(-1.34)</td>
<td>(-1.45)</td>
<td>(-1.43)</td>
<td>(-1.00)</td>
<td>(-0.79)</td>
<td>(-0.62)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Q ZLB</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>10Y</td>
</tr>
<tr>
<td>B</td>
<td>-0.04*</td>
<td>-0.04*</td>
<td>-0.04*</td>
<td>-0.04*</td>
<td>-0.04*</td>
<td>-0.04*</td>
</tr>
<tr>
<td>Z-score</td>
<td>(-1.85)</td>
<td>(-1.85)</td>
<td>(-1.85)</td>
<td>(-1.84)</td>
<td>(-1.84)</td>
<td>(-1.84)</td>
</tr>
<tr>
<td></td>
<td>3M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>10Y</td>
</tr>
<tr>
<td>B</td>
<td>-0.31***</td>
<td>-0.30***</td>
<td>-0.27***</td>
<td>-0.22***</td>
<td>-0.19***</td>
<td>-0.16***</td>
</tr>
<tr>
<td>Z-score</td>
<td>(-3.83)</td>
<td>(-3.82)</td>
<td>(-3.75)</td>
<td>(-3.52)</td>
<td>(-3.24)</td>
<td>(-3.06)</td>
</tr>
<tr>
<td></td>
<td>3M</td>
<td>1Y</td>
<td>3Y</td>
<td>6Y</td>
<td>8Y</td>
<td>10Y</td>
</tr>
<tr>
<td>B</td>
<td>-0.24</td>
<td>-0.26</td>
<td>-0.29**</td>
<td>-0.30***</td>
<td>-0.28***</td>
<td>-0.26***</td>
</tr>
<tr>
<td>Z-score</td>
<td>(-1.21)</td>
<td>(-1.47)</td>
<td>(-2.13)</td>
<td>(-2.84)</td>
<td>(-3.03)</td>
<td>(-3.04)</td>
</tr>
</tbody>
</table>
Table 3.7: Factor Loadings for Switzerland. The table reports the values of the reserves factor loadings for the pre-ZLB period (upper panel) and ZLB period (lower panel) for selected yields, together with the Z-score from the Z-test (in parenthesis). The levels of significance of 0.1, 0.05 and 0.01 are denoted with *, ** and ***, respectively.

<table>
<thead>
<tr>
<th></th>
<th>pre-ZLB</th>
<th>ZLB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3M 1Y 3Y 6Y 8Y 10Y</td>
<td>3M 1Y 3Y 6Y 8Y 10Y</td>
</tr>
<tr>
<td>B</td>
<td>Q</td>
<td>Q</td>
</tr>
<tr>
<td></td>
<td>-0.00 -0.00 -0.00 -0.00 -0.00 -0.00</td>
<td>-0.04*** -0.04*** -0.04*** -0.03*** -0.03*** -0.03***</td>
</tr>
<tr>
<td>Z-score</td>
<td>(-0.72) (-0.72) (-0.72) (-0.72) (-0.72) (-0.72)</td>
<td>-(2.64) -(2.65) -(2.66) -(2.67) -(2.68) -(2.68)</td>
</tr>
<tr>
<td>level risk</td>
<td>3M 1Y 3Y 6Y 8Y 10Y</td>
<td>3M 1Y 3Y 6Y 8Y 10Y</td>
</tr>
<tr>
<td>B</td>
<td>0.01 0.01 0.01 0.00 0.00 -0.00</td>
<td>-0.08*** -0.08*** -0.08*** -0.08*** -0.07*** -0.07***</td>
</tr>
<tr>
<td>Z-score</td>
<td>(1.56) (1.50) (1.28) (0.77) (0.21) (-0.67)</td>
<td>-(2.59) -(2.63) -(2.75) -(2.90) -(2.96) -(2.97)</td>
</tr>
<tr>
<td>level &amp; slope risk</td>
<td>3M 1Y 3Y 6Y 8Y 10Y</td>
<td>3M 1Y 3Y 6Y 8Y 10Y</td>
</tr>
<tr>
<td>B</td>
<td>-0.01 -0.01 -0.01 -0.01 -0.01 -0.01</td>
<td>0.03 0.00 -0.04 -0.06* -0.07** -0.07**</td>
</tr>
<tr>
<td>Z-score</td>
<td>(-0.24) (-0.26) (-0.36) (-0.70) (-1.01) (-1.33)</td>
<td>(0.40) (0.08) -(0.82) -(1.84) -(2.16) -(2.26)</td>
</tr>
</tbody>
</table>
Figure 3.1: *Interest Rates and Central Bank Reserves in the US.* The term structure of interest rates (left scale) is plotted for the non-ZLB (upper panel) and ZLB-period (lower panel), together with the NBR (right scale).
Figure 3.2: *Interest Rates and Central Bank Reserves in Switzerland*. The term structure of interest rates (left scale) is plotted for the non-ZLB (upper panel) and ZLB-period (lower panel), together with the total sight deposits (right scale).
Figure 3.3: Estimated Posteriors for the US. The Figure illustrates the estimated posterior densities for all the parameters of the model used to estimate the QE effect in US data.
Figure 3.4: Latent Factors for US Data. The Figure plots the evolution of the three estimated latent factors in the period away from the ZLB (upper panel) and at the ZLB (lower panel) for US data.
Figure 3.5: Cumulative Effects for the US. The figure plots the cumulative effects over time of increases in NBR on the 3-month (left-hand side) and the 10-year (right-hand side) interest rates according to the $Q$-model (top), the $l$-model (middle) and $l&$s-model (bottom) together with 90% credible intervals. The credible interval is calculated using the posterior parameters’ distributions, i.e. every 1000th set of parameters along the first Markov chain and after burn-in.
Figure 3.6: *Cumulative Effects for Switzerland.* The figure plots the cumulative effect over time of increases in total sight deposits at the SNB on the 3-month (left-hand side) and 10-year (right-hand side) interest rates according to the $Q$-model (top), the $l$-model (middle) and $l&s$-model (bottom) together with the 90% credible interval. The credible interval is calculated using the posterior parameters’ distributions, specifically every 1000th set of parameters along the first Markov chain and after burn-in.
Figure 3.7: *Separating Supply and Liquidity Effects in US Data.* The upper panel depicts weekly data of NBR and the Federal Reserves’ holdings of Treasury securities with more than one year to maturity. The lower panel draws the cumulative liquidity effect by multiplying the difference in the changes of reserves and Treasury holdings by the NBR factor loadings of the 10-year maturity as estimated in Table 3.6. The solid black line refers to the most conservative estimate at the ZLB which results under the Q-measure, the dashed red line to the most optimistic estimate that results from the $l & s$ model.
Bibliography


120


Curriculum Vitae

Personal Details

Name          Barbara Caroline Sutter
Date of Birth 27 February 1980
Citizenship   Swiss (Ormalingen and Winterthur)

Education

2008 - 2013 University of St. Gallen, Switzerland
PhD in Economics and Finance

2008    Study Center Gerzensee, Switzerland
Program for Beginning Doctoral Students in Economics

2005 - 2007 University of St. Gallen, Switzerland
Master of Arts in Quantitative Economics and Finance, M.A. HSG

2001 - 2004 University of St. Gallen, Switzerland
Bachelor of Arts in Economics, B.A. HSG

Professional Experience

2011 - Swiss National Bank, Zurich, Switzerland
Money Market and Foreign Exchange

2007 - 2010 Swiss National Bank, Zurich, Switzerland
Economic Research

2005 UBS AG, Zurich, Switzerland
Wealth Management International, Active Portfolio Supervision

2004 UBS AG, Zurich, Switzerland
Wealth Management International, Product & Sales Management BCG