Optimal Policy in Models of Search Unemployment with Frictions and Externalities

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The President:

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<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>vii</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.D Implementation by wage dependent benefits</td>
<td>123</td>
</tr>
<tr>
<td>3.E Tables</td>
<td>124</td>
</tr>
<tr>
<td>Bibliography</td>
<td>125</td>
</tr>
<tr>
<td>Curriculum Vitae</td>
<td>136</td>
</tr>
</tbody>
</table>
## List of Figures

1.1 Uncompensated comparative statics ........................................ 21
1.2 The transition flows ......................................................... 27
1.3 First best implementations in the intergroup model .................. 35
2.1 Cumulative distribution functions - mean preserving single crossing spread ...................................................... 86
2.2 Probability density functions - mean preserving single crossing spread ................................................................. 87
3.1 Optimal unemployment benefits over the business cycle for different degrees of returns to scale .............. 109
# List of Tables

1.1 Decentralized economy, benchmark .......................... 33  
1.2 Social planner’s solution ........................................... 34  
1.3 Effect of a low wage subsidy on unemployment rates ........ 36  
1.4 Variable names ................................................... 50  
2.1 Numerical example .................................................. 86  
3.1 Empirical evidence on matching elasticities ...................... 92  
3.2 Benchmark calibration ............................................... 113  
3.3 Optimal unemployment benefits over the cycle ................. 113  
3.4 Different calibration scenarios ..................................... 124  
3.5 Optimal unemployment benefits for different marginal product assumptions ......................................................... 124
Summary

My thesis consists of three chapters, all dealing with the characterization of optimal policy for different extensions of the canonical Diamond-Mortensen-Pissarides model.

Chapter 1 characterizes optimal labor market policy in a potentially turbulent environment. A fiscal externality, generated by the existence of a partial unemployment insurance system, distorts the pre-policy equilibrium along three margins: job creation, job acceptance, and job destruction. Optimal policy is characterized by a payroll tax, a firing tax, and a hiring subsidy. It is further shown that the derived policy mix is robust to the introduction of economic turbulence in form of state-dependent worker transitions between skill classes. This is crucial as widely discussed intergroup redistribution schemes, like in-work benefits targeted at low-skilled workers, are rendered considerably less effective in that case.

Chapter 2 embeds search unemployment in a North-North intermediate goods trade framework. International product market integration leads to redistribution of market shares from ‘weak’ to ‘strong’ firms within an industry, implying chances and threats for firms at the same time, as firms are ex-ante unaware of their relative advantage over the competitor. Opening the economy will therefore increase the dispersion of potential revenues and consequently lead to higher labor market turnover and higher welfare, while the effect on employment is ambiguous. Ceteris paribus, the effects are qualitatively similar to reducing employment protection. It is shown that the positive welfare effects of opening to trade are decreasing in the level of firing protection, which can therefore prevent an economy from reaping the full benefits of trade liberalization.

Chapter 3 generalizes the matching function to allow for non-constant returns to scale which induces non-optimal levels of aggregate search. A generalized Hosios-condition is derived that not only determines the split of the match surplus but also its optimal size. Unemployment benefits can be used as a Pigouvian instrument to restore efficiency. The implementation of the optimal
allocation involves pro-cyclical (counter-cyclical) benefits correcting for excessive (too little) aggregate search activity if the matching function exhibits decreasing (increasing) returns to scale.
Zusammenfassung

Meine Dissertation besteht aus drei Kapiteln, welche alle die Charakterisierung optimaler Arbeitsmarktpolitik für verschiedene Erweiterungen des Diamond-Mortensen-Pissarides Modells zum Thema haben.


Kapitel 3 verallgemeinert die Matchingfunktion, sodass diese auch nicht-kons-
Introduction

This thesis combines three papers that are diverse with respect to their research questions. However, all share a similar normative approach using different extensions of the canonical model of search unemployment pioneered by Diamond, Mortensen and Pissarides (DMP). Those extensions give rise to additional inefficiencies beyond the typical search externalities that require richer forms of policy intervention. In a nutshell, the basic idea that distinguishes the DMP model from its neoclassical counterpart is that both workers and firms have to find each other and that this form of search is time-consuming due to informational and other imperfections. This friction is usually modeled using an aggregate matching technology that converts number of job searchers $U$ and number of vacancies $V$ into matches $M = m(U, V)$. The probabilities for a worker to find a job and for the firm to find a worker therefore depend on the ratio of the aggregates $V$ and $U$. Workers and firms typically take those probabilities as given and do not internalize the effect on other agents in their individual decision making. As search takes time agents do not meet in one market all at once, as in the neoclassical benchmark, but in pairs. Once matched, the firm and the worker bilaterally bargain over a surplus which arises because waiting for another partner is costly. Models with labor market search frictions allow addressing many issues like equilibrium unemployment, explicit worker and job flows, wage dispersion for homogeneous workers (Burdett and Mortensen, 1998), business cycle dependence of worker flows (Mortensen and Pissarides, 1994), evaluation of policies targeted at the hiring or firing margin (Mortensen and Pissarides, 2003), etc. that cannot be explained in the neoclassical benchmark. More extensive summaries are provided by the Nobel Laureate lectures in Diamond (2011), Mortensen (2011), and Pissarides (2011).

The question of optimality in DMP-type models is the main focus of the thesis. In the standard DMP model the decentralized economy is characterized by two forms of externalities that arise from the nature of time-consuming search. First, there is a negative within-group externality. While the search activity of a
single worker has a positive influence on his own probability of finding a job he
does not internalize that this deteriorates the chances of other workers. A simi-
lar argument can be stated for firms. Second, there is a positive between-group
externality. The search activity of a single worker raises the chances for firms
to match with a worker and vice versa. Comparing the decentralized economy
allocation with the allocation a benevolent social planner would choose reveals
that there is an optimal trade-off between the negative and the positive exter-
nality. This is characterized by a simple rule that relates the way the match
surplus is split according to bilateral bargaining to the sensitivity of the indi-
vidual matching probabilities to changes in the aggregates \( U \) and \( V \), measured
by the elasticities of the matching function. This rule is known as the Hosios
(1990)-condition.

Every chapter of my thesis develops a model that introduces additional fric-
tions on top of the search externalities present in the canonical version of the
DMP-model. The first paper adds a fiscal externality and characterizes optimal
policy in a framework with policy spill-overs generated by employment-state
dependent accumulation and decumulation of human capital. The second pa-
er introduces a novel way of incorporating international trade and looks at
the interaction of a reduction in trade barriers or openness to trade and em-
ployment protection legislation. In the third paper I relax the, in theory widely
used but empirically hardly supported, assumption of constant returns to scale
of the matching function which alters the typical efficiency results and con-
sequently optimal policy. All chapters are single-authored and self-contained
such that they can be read independently of each other. In the rest of this intro-
ductory section I summarize the key results of the three papers in more detail.

The first chapter addresses the question of policy targeting. Most scholars agree
that unemployment rates in the OECD, especially of low-skilled workers, are
excessively high, but there is little consensus on what should be done about it,
leading to a variety of policy advice. While there exists much empirical work
focusing on the relative effectiveness of specific instruments, the aim of this
paper is to develop a theoretical characterization of optimal policy. I employ
a dynamic extended DMP model, rich enough to incorporate three important
decision margins: job creation, job acceptance, and job destruction. A firing ex-
ternality, generated by the existence of a partial unemployment insurance sys-
tem, distorts the pre-policy equilibrium along those three margins and leads to
excessive unemployment. Optimal policy is characterized by a payroll tax, a fir-
ing tax, and a hiring subsidy. Endogenous job acceptance demands that a firing tax and a hiring subsidy have to be set equal in any case and cannot be used to correct for the possible failure of the Hosios condition. In that case the optimal policy mix has to be extended by either an output or a recruitment tax/subsidy. The derived policy mix contrasts the results of Blanchard and Tirole (2008) who, in a static approach neglecting job creation and acceptance, characterize optimal policy by redistribution from firing firms to unemployed workers, which resembles the experience rating system implemented in the United States. An important finding is that my characterization of optimal policy is robust to the introduction of economic turbulence, modeled as employment-state-dependent transitions of workers between skill-classes, i.e. learning-by-doing on the job versus skill loss during unemployment. This is crucial as a lot of existing policy advice is rendered considerably less effective in that case. The cross-financed wage subsidy scheme for low-skilled workers as proposed by Mortensen and Pissarides (2003) is an example for such a policy advice. They assume skill classes to operate in complete juxtaposition, except for the connection via the government’s budget constraint, which underestimates the adverse effect on high-skilled workers who become pickier concerning their acceptance and continuation decisions as their fall back option increases with the wage subsidy for low-skilled workers. To summarize, instead of redistribution from high-to low-skilled workers or from firing firms to unemployed workers, the paper identifies a scheme involving redistribution from firing to hiring firms to be optimal.

In the second chapter I contribute to the strand of the literature that emphasizes the need for understanding the role of social protection in a world of increasing economic integration. The main focus has been put on unemployment insurance (UI) and unemployment levels while employment protection (EP) and labor market turnover has received comparatively little attention in international trade. I therefore present a model that is specifically designed to analyze the effect of trade on worker flows and therefore labor market turnover, which is the margin where EP is most likely to have an effect on. The framework consists of a parsimonious DMP model with endogenous separation that is embedded in a North-North intermediate goods trade setting. International product market integration affects marginal revenues - which are typically constant in the canonical DMP setting - and leads to redistribution of market shares from ‘weak’ to ‘strong’ firms within an industry, implying chances and threats for firms at the same time, as firms are ex-ante unaware of their relative ad-
vantage over the competitor. Opening the economy will therefore increase the dispersion of potential revenues and consequently lead to higher labor market turnover, higher welfare and increased wage inequality, while the effect on employment is ambiguous. Ceteris paribus, the effects are qualitatively similar to decreasing EP in form of costly firing restrictions which prevent the economy from reaching a first best allocation. The positive welfare effects of opening to trade are decreasing in the level of firing costs. This can therefore lead to a substantial failure in reaping the benefits that could result from economic integration, by preventing labor reallocation. These main results are robust to the introduction of risk-averse workers. The paper connects to different strands of the literature. First, while the idea that international integration should lead to more volatility in employment has been raised before (see e.g. Rodrik, 1997 or Bhagwati and Dehejia, 1994) it has hardly been formalized and micro-founded like in this paper. Second, my technology assumptions depart from the typical Melitz (2003)-type models that neglect import competition due to assuming constant elasticity of substitution. I adopt a similar idea to trade in tasks by Grossman and Rossi-Hansberg (2008) that emerged from the new offshoring literature. Using non-homothetic technology with more elaborate substitution characteristics also creates more complex competition patterns.

The third chapter investigates the role of unemployment benefits (UB) as a Pigouvian instrument in contrast to their typical function of insurance provision for workers against the risk of unemployment. The crux is that UB can be used to correct socially inefficient search behavior. Hence, there are situations when UB should be non-zero even if workers are risk-neutral. When the magnitude of distorted aggregate search depends on the state of the economy along the business cycle, unemployment benefits optimally have to vary over the cycle too. The paper presents a DMP model that explicitly allows for non-constant returns to matching that generate this type of externalities without the empirically hardly justifiable assumption of sticky wages for new hires, which is done in related studies, e.g. Landais et al. (2010). It is shown that without policy intervention a generalized Hosios condition guaranteeing constrained efficiency cannot hold in any state of the economy. The implementation of the optimal allocation involves pro-cyclical (counter-cyclical) benefits correcting for excessive (too little) aggregate search activity if the matching function exhibits decreasing (increasing) returns to scale. The classical Hosios condition balances the typical search externalities in an efficient way by determining the optimal split of the match surplus. The generalized Hosios condition in addition also
corrects the size of the surplus which is non-optimal if the returns to scale of the matching function are non-constant. Numerical simulations suggest that the role of cycle-dependent Pigouvian benefits is quantitatively rather limited compared to the typical function of insurance provision. This conclusion, however, is directly related to the Shimer (2005a)-puzzle and the implausibly low responsiveness of the match surplus to productivity shocks for this class of models.
Chapter 1

Labor Market Policy Instruments and the Role of Economic Turbulence

Philip Schuster†

†The paper was presented at seminars at the University of St. Gallen, the IZA summer school (2010), the Gerzensee summer course (2010), the annual conference of the Association for Public Economic Theory (2010, Istanbul) and the annual meeting of the German Economic Association (2010, Kiel). I thank the participants for helpful comments and discussions. In particular I thank Pierre Cahuc, Carlos Carrillo-Tudela, Christian Haefke, Gabriel Felbermayr, Reto Föllmi, Christian Keuschnigg, Jochen Mankart, Jean-Baptiste Michelau, Michael Reiter, Evelyn Ribi, Uwe Sunde, and Iván Werning for their advice. All remaining errors are my own.
1.1 Introduction

Times of high unemployment always inspire debates on the role of labor market policy. While most scholars agree that unemployment rates in the OECD, especially of low-skilled workers, are excessively high, there is little consensus on what should be done about it, leading to a variety of policy advice. Take for example the recent debate in Germany on the controversial subject of which of two specific policy instruments should be introduced: wage subsidies (e.g. Sinn et al., 2006) versus hiring subsidies (e.g. Brown et al., 2007). While there exists much empirical work focusing on the relative effectiveness of specific instruments, the aim of this paper is to develop a theoretical characterization of optimal policy. I will employ a dynamic Diamond-Mortensen-Pissarides (DMP) model, rich enough to incorporate three important decision margins: job creation, job acceptance, and job destruction, in a potentially turbulent environment. Turbulence is introduced to the model, in the spirit of Ljungqvist and Sargent (1998), as state-dependent transitions of workers between skill classes, i.e. unemployed workers lose their skills in the course of time. This adds an additional channel for policy spill-over to the framework which plays an important role in case of policy targeting. Possible inefficiencies of the initial, pre-policy equilibrium come in the form of a typical search externality and a firing externality, which I will focus on in more detail. The need to finance an existing partial unemployment insurance (UI) system, not internalized by firms, gives rise to the latter externality. I introduce a set of policy instruments: payroll, output and firing taxes as well as wage, hiring and recruitment subsidies, while I do not allow for taxation of unemployed workers. The analysis shows that the optimal policy mix can be characterized by a payroll tax as a fiscal instrument set to finance the UI system and a ‘firing tax equal hiring subsidy’-scheme similar to Ligthart and Heijdra (2000) and Heijdra and Ligthart (2002), representing redistribution from firing to hiring firms, to correct for the distortions\(^1\). This contrasts the results of Blanchard and Tirole (2008) (henceforth BT08) who, in a static approach, neglecting job creation and acceptance, characterize optimal policy by redistribution from firing firms to unemployed workers, which resembles the experience rating system implemented in the United States\(^2\). The derived optimal policy mix has to be expanded by either an output or a recruitment tax/subsidy in case of ‘unbalanced’ search externalities to correct for the

\(^1\)In a recent paper, Ricci and Waldmann (2011) derive a similar policy recommendation to correct for a hold-up problem caused by contractual incompleteness concerning firm-specific training. In contrast, in my paper workers’ skills are general and not attached to specific firms.

\(^2\)The fundamental insights were first described by Feldstein (1976) and Topel (1983).
1.1. INTRODUCTION

failure of the Hosios (1990)-condition. An important finding is that the charac-
terization of optimal policy is robust to the introduction of economic turbulence.
This is crucial as a lot of existing policy advice is rendered considerably less
effective in that case. The cross-financed wage subsidy scheme for low-skilled
workers, representing redistribution from high- to low-skilled workers, as pro-
posed by Mortensen and Pissarides (2003) (henceforth MP03) is an example for
such a policy advice. They assume skill classes to operate in complete juxtapo-
sition, except for the connection via the government’s budget constraint, which
underestimates the adverse effect on high-skilled workers who become pickier
concerning their acceptance and continuation decision as their fall back option
increases with the wage subsidy for low-skilled workers. In addition the skill
composition deteriorates as a result of such a targeting scheme.

This paper relates to several strands of the literature. As mentioned above there
are numerous empirical studies\(^3\) that deal with estimating the employment
effects of various policy instruments. But typical inference from comparing
treated and untreated individuals to evaluate ‘big scale’ policies faces the fol-
lowing problems. First, a policy has actually to be in place. Second, ‘big scale’
policies will induce general equilibrium effects which lead to a violation of the
necessary ‘stable unit treatment value assumption’ (see Angrist et al., 1996).
In their study on counseling, Cahuc and Barbanchon (2010) argue how micro
evaluations neglecting crowding out, adverse spill-over effects on non-targeted
persons, and other equilibrium effects can lead to misguided policy advice. I
therefore base my analysis on a model of equilibrium search unemployment
rich enough to capture those effects. Dynamic DMP models have been widely
used to evaluate different labor market policy instruments ever since the influ-
ential paper of Mortensen and Pissarides (1994). However, the conclusions so
far are mixed. While Bovenberg et al. (2000) and Cardullo and van der Lin-
den (2006) argue that wage subsidies can substantially reduce unemployment,
Boone and van Ours (2004) and Oskamp and Snower (2008) find no such effect.
What these and similar studies typically have in common is that they embed
the frictional labor market in complex CGE-models which makes it hard to dis-
entangle different effects or draw conclusions concerning the optimal design
of policy which is at the heart of my paper. A more theoretical treatment -
probably most closely related to this paper - is provided by MP03. They an-

\(^3\)Empirical evidence on the effects of wage subsidies is summarized by Katz (1996) for the United
States, Bell et al. (1999) for the United Kingdom, and Bonin et al. (2002) for Germany. Boockmann et al.
(2007) provide some evidence on ‘hiring subsidy’-like grants in Germany. A review on the empirical
effects of employment protection is presented in Lazear (1990) and Skedinger (2011).
alytically derive optimal policy before presenting some simulation results for non-optimal policy schemes. I will extend their analysis on several dimensions. First, in their optimal policy characterization, MP03 solely concentrate on the distorting effects of subsidies and taxes while fiscal effects are suppressed by allowing for non-distortionary consumption taxes. Hence, the firing externality in the spirit of BT08 does not play a role in their setting. Second, I introduce an additional margin, namely job acceptance, which will alter optimal policy if search externalities are ‘unbalanced’. Third and most importantly, I introduce economic turbulence and discuss its role for policy design. Another closely related paper is Michau (2011) who extends the BT08-setting to a dynamic DMP framework. He explicitly models the UI problem with risk-averse workers and finds that the welfare maximizing allocation is characterized by full insurance and output maximization. As Nash bargained wages with positive bargaining power of the workers are incompatible with full insurance, he finds that in a second best a social planner would reduce labor market tightness, implemented by a positive spread between a firing tax and a hiring subsidy. This is done to reduce wages and therefore decrease under-provision of UI. I do not consider this trade-off between insurance and output maximization here by conditioning my analysis on the existence of a partial UI system as the focus lies on role of economic turbulence in designing policy. The idea of economic turbulence is inspired by work of Ljungqvist and Sargent (1998). While this strand of the literature, including Pissarides (1992), Ljungqvist and Sargent (2004), and Den Haan et al. (2005), is concerned with the influence of skill depreciation during unemployment on the persistence of unemployment, its implication for policy design has received little attention so far.

The outline of the paper is as follows. First, a simple intragroup model is developed featuring only one skill class. The optimal policy mix, implementing the social planner’s solution, is characterized, which will provide good intuition for the more complex intergroup model discussed in section 1.3. I extend the intragroup by an additional skill class and allow for redistribution as well as economic turbulence in form of state-dependent transitions of workers between the skill classes. After showing how the characterization of optimal policy in the intergroup model relates to the intragroup case I present some simulation results in order to highlight also the quantitative dimension of my findings.
1.2 A simple intragroup model

The model is based on the standard dynamic Mortensen and Pissarides (1994)-framework enriched by endogenous acceptance. In this section I consider only one skill class. There are two types of rational, forward looking agents: workers and firms. Labor force $L$ is comprised of atomistic risk-neutral workers. The assumption of risk-neutrality is discussed more thoroughly in section 1.2.2. There is a sufficiently large number of risk-neutral firms that can enter the labor market instantaneously but are subject to per-period net flow costs $c$ for posting a vacancy. For production each firm needs one worker who will inelastically supply one unit of labor if employed. The three decision margins: job creation ($\theta$), job acceptance ($x$), and job destruction ($\hat{x}$) are best understood when looking at the life cycle of a job. First, firms decide to post vacancies according to a free entry condition which fixes labor market tightness $\theta$. The search friction implies that it takes time to fill a vacancy during which a firm has to pay per-period gross posting costs $C$ that are reduced by a recruitment subsidy $R$ to $c = C - R$. Eventually a worker and a firm are matched according to a matching technology $m$. This can be interpreted as meeting for a job interview. Only then the agents will learn how well suited an applicant is for the specific job. This is modeled as drawing a job-specific productivity $x$ from a known distribution $G(\cdot)$. If the realization of the draw is higher than the according reservation productivity, referred to as ‘outside’ cut-off, i.e. $x > \bar{x}$, the job is started and the firm receives a one-time hiring subsidy $H$. Technically, this is one of the main difference compared to MP03 who assume that every job is created at maximum idiosyncratic productivity, trivializing the acceptance decision because job offers are rejected with probability zero. During production the firm receives net off tax output $(1 - \tau)x$ and an in-work benefit or wage subsidy $D$ that partly compensates for the wage $w$, stemming from a Nash bargaining game, it has to pay to the worker. A new idiosyncratic productivity shock arrives with probability $\pi$. If a new draw is lower than the endogenous ‘inside’ cut-off, i.e. $x < \hat{x}$, the job is destroyed and the firm has to pay a separation or firing tax $F$. To summarize the featured instruments. Three different subsidies will be analyzed: a periodic lump-sum wage subsidy ($D$), a one-time hiring subsidy to the firm ($H$), and a recruitment subsidy ($R$). On the other hand I will analyze three distortionary taxes, namely: firing taxes ($F$),

\footnote{In an alternative interpretation this relates to Hall (2005) who also allows for less qualified persons to apply. In contrast to my analysis, he assumes that the qualification of an applicant is not completely revealed to the employer in the first meeting. This can only be resolved if the employer decides to costly evaluate the application.}
linear output taxes ($\tau$), and linear\(^5\) payroll taxes ($t$). An important assumption I make is that unemployed workers cannot be taxed. Part of the value of non-work is home production which cannot be transformed into tax revenue. This rules out non-distortionary consumption taxation. Analytically, the model can be described as follows.\(^6\)

As usual for this kind of framework an aggregate matching function \(M(u,v)\), which maps the stock of unemployed \((u)\) and the stock of vacancies \((v)\) into the flow of new matches \((M)\), is assumed to be homogeneous of degree one with elasticity w.r.t. \(u\) of \(0 < \eta < 1\). Defining labor market tightness as \(\theta \equiv \frac{v}{u}\) results in the matching probability functions (1.1) and (1.2) for firms and workers,

\[
\begin{align*}
\text{prob. of a match for the firm:} & \quad \frac{M(u,v)}{v} = q(\theta), \\
\text{prob. of a match for the worker:} & \quad \frac{M(u,v)}{u} = \theta q(\theta),
\end{align*}
\]

with \(q'(\cdot) < 0\), \(q''(\cdot) < 0\) and \(M(u,v) \leq \min\{u,v\}\). Further define \(q^f \equiv q(\theta)(1 - G(x))\) and \(q^w \equiv \theta q(\theta)(1 - G(\hat{x}))\) as the joint probabilities of matching and accepting. A worker can be either employed \((e)\) or unemployed \((u)\), that is I abstract from transitions into and out of labor force, hence \(e + u = L\). Each state is associated with a specific present value, \(U\) for being unemployed and \(W(x)\) or \(\hat{W}(x)\) for becoming or being employed, respectively. A firm participating in the labor market can be in two states. Either it is looking for a worker which has value \(V\) or it is employing a worker which gives \(J(x)\) or \(\hat{J}(x)\). In general, the hat-notation always indicates that the worker or the firm have already been in the same state before the arrival of a shock. Or put differently, ‘without hat’ can be referred to as the initial or ‘outside’ value while ‘with hat’ denotes the continuation or ‘inside’ value. Given the assumption of perfect capital markets, where \(r\) denotes the exogenous interest rate, I can write both asset equations of working as follows

\[
\begin{align*}
(1-t)w(x) & + \pi' U - W(x), \\
(1-t)\hat{w}(x) & + \pi' \hat{U} - \hat{W}(x).
\end{align*}
\]

\(^5\)The linearity assumption does not drive the fundamental results but helps to keep the mathematics straightforward. Note that the lump-sum component of the wage subsidy going to the worker and the linear component \(t\) can mimic a regressive or progressive tax schedule. The implementation of the efficient allocation, as derived in section 1.2.2, does not require a wage subsidy.

\(^6\)The notation is based on Pissarides (2000) with few exceptions. A description of all used variables can be found in appendix section 1.G.
A just recently employed worker’s felicity equals after-tax wage income \((1 - t)w(x)\) or \((1 - t)\hat{w}(x)\), respectively. When a shock arrives he loses \(W(x)\) and gains \(U\) if the new productivity draw \(x\) is lower than the ‘inside’ cut-off \(\hat{x}\), hence with probability \(G(\hat{x})\). With probability \((1 - G(\hat{x}))\) he gets \(\hat{W}e\), which denotes the conditional expectation\(^7\) of the value of being employed. The asset value of being unemployed is given by

\[
rU = z + q^w (W^e - U),
\]

where \(z\) denotes the value of non-work which is composed of unemployment compensation \(b\) and home production \(h\) in a linear way, \(z \equiv b + h\). Turning to the firms’ side the asset value of a vacancy can be written as

\[
rV = -c + q^f (f^e + H - V), \quad \text{where} \quad c = C - R. \tag{1.6}
\]

Two subsidies enter this relationship. In case of an accepted match the firm has to give up the value of a vacancy \(V\) but gets the expected value of a job for the firm \(f^e\) plus a hiring subsidy \(H\). The gross flow costs of maintaining a vacancy \(C\) minus the recruitment subsidy \(R\) give the net costs \(c\). As free entry is imposed and \(V\) is decreasing in \(\theta\), in equilibrium \(V\) is driven down to zero which will pin down \(\theta\), hence

\[
V = 0 \Rightarrow \theta. \tag{1.7}
\]

The asset values of a job are given in (1.8) and (1.9),

\[
rJ(x) = (1 - \tau)x - w(x) + D + \pi^n \left[ (1 - G(\hat{x}))\hat{f}^e - G(\hat{x})F - J(x) \right], \tag{1.8}
\]

\[
r\hat{f}(x) = (1 - \tau)x - \hat{w}(x) + D + \pi^n \left[ (1 - G(\hat{x}))\hat{f}^e - G(\hat{x})F - \hat{f}(x) \right]. \tag{1.9}
\]

In the current period a firm receives after-tax\(^8\) production \((1 - \tau)x\) minus wage rate \(w(x)\) or \(\hat{w}(x)\) plus a wage subsidy \(D\).\(^9\) In case of a separation, which occurs with probability \(\pi^n\) and the probability of \(x < \hat{x}\), a firm has to pay a firing tax \(F\). Observe that given the wage determination explained below a firm and a

---

\(^7\)The conditional expectation of some random variable \(X(x)\) w.r.t. \(\hat{x}\) is defined as \(E(X(x)|x > \hat{x}) \equiv X^e = \int_{\hat{x}}^{\infty} \frac{X(x)}{1 - G(\hat{x})} dG(\hat{x})\). Note the difference in notation compared to \(E(X(x)|x > \hat{x}) \equiv X^e\).

\(^8\)One might argue that an output tax is a rather abstract instrument in contrast to for example a cash-flow tax. Note however that cash-flow taxedation of form \(\tau [x - w(w)]\) would just imply a mixture of output taxation and subsidization of wage costs. Later is not directly implemented in the model, but can be mimicked by adjusting the employee’s wage tax \(t\) because of Nash bargaining.

\(^9\)Note that with Nash bargaining it does not matter economically whether the wage subsidy is given to the worker or the firm but the interpretation of \(w\) changes. In my setting \(w\) and \((1 - t)w\) are interpreted as gross and net wages received by the worker already including all subsidies.
worker will always mutually agree to destroy or create a job, i.e. both sides have the same reservation productivities. Hence, the notions of a ‘firing’ and a ‘separation’ tax are equivalent. The reservation productivities are pinned down by the following conditions

\[ J(x) + H = 0 \Rightarrow \tilde{x}, \]  
\[ \hat{J}(\hat{x}) + F = 0 \Rightarrow \hat{x}. \]  

The first relation states that after meeting for an interview and observing the match specific productivity \( x \), a job will only be generated if the value of a job including the one-time hiring subsidy is non-negative. The second condition reflects that a firm will only want to continue a job if its value covers at least the firing tax. Wages are determined via Nash bargaining and are renegotiated every time a shock arrives. The Nash wages are given as solutions to the following optimization problems, where the weight \( \omega \) can be interpreted as the worker’s bargaining power,

\[ w(x) = \arg\max (W(x) - U)^\omega (J(x) + H)^{1-\omega}, \]  
\[ \hat{w}(x) = \arg\max (\hat{W}(x) - U)^\omega (\hat{J}(x) + F)^{1-\omega}. \]  

**Lemma 1.2.1.** If \( F = H \) then \( w(x) = \hat{w}(x) \).

**Proof.** Note that \( w(x) = \hat{w}(x) \) implies \( W(x) = \hat{W}(x) \) and \( J(x) = \hat{J}(x) \) by construction. If \( F = H \), the two problems (1.12) and (1.13) are identical.

The result of this lemma is more general and can be extended to non-linear utility and non-linear wage income taxation\(^{10}\). Given my assumptions the equilibrium ‘outside’ and ‘inside’ wage rates can be solved for explicitly\(^{11}\)

\[ w(x) = (1 - \omega) \frac{z}{1 - t} + \omega((1 - \tau)x + D + c\theta + rH) - \omega \pi^n(F - H), \]  
\[ \hat{w}(x) = (1 - \omega) \frac{z}{1 - t} + \omega((1 - \tau)x + D + c\theta + rF). \]  

Observe that the ‘inside’ and ‘outside’ wage distributions are directly related to the productivity distribution \( G(\cdot) \) for \( x \) larger than the respective cut-off. A wage subsidy \( D \) will increase both wage schedules by the share the worker can claim in the process of bargaining \( \omega D \). While a recruitment subsidy \( R \), which is included in \( c \), decreases both wages to the same extent, they respond

\(^{10}\)Insert \( u(w(x) - T(x)) \) in (1.3) and \( u(\hat{w}(x) - T(x)) \) in (1.4), with the mild conditions \( u'(\cdot) > 0 \) and \( w(x) - T(x) > 0 \) for otherwise arbitrary functions \( u(\cdot) \) and \( T(\cdot) \). The proof still holds.

\(^{11}\)See appendix section 1.C for the derivation.
differently to a hiring subsidy $H$ and a firing tax $F$. A hiring subsidy will increase the ‘outside’ wage of a worker while it does not affect the ‘inside’ wage as the subsidy is already sunk. A firing tax will abate ‘outside’ wages as firms are more cautious about hiring workers because they eventually have to pay $F$. In contrast, ‘inside’ wages will be inflated by $F$ because firms are more willing to hold on to workers once they are employed. The relationship of ‘outside’ and ‘inside’ wage is simply $w(x) = \hat{w}(x) - (r + \pi^n)\omega(F - H)$. At last, in equilibrium the government’s budget constraint has to hold,

$$0 = (L - u)\bar{w}t + (L - u)\bar{x}\tau + (L - u)\pi^nG(\hat{x})F - uq^wH - \theta uR - (L - u)D - ub. \quad (1.16)$$

where $\bar{w}$ and $\bar{x}$ denote average wage and productivity, respectively. The first line represents tax income from the payroll tax, the output tax and the firing tax. The second line gives expenditure on hiring, recruitment, and wage subsidies as well as unemployment benefits.

### 1.2.1 Equilibrium

The equilibrium vector $\langle u, \theta, x, \hat{x} \rangle$ is pinned down by the four equations (1.17) to (1.20)$^{12}$. Equilibrium is partly recursive, i.e. only (1.17) and (1.18), henceforth referred to as the JD-JC system, have to be solved simultaneously for $\theta$ and $\hat{x}$ after inserting (1.19). The job creation (JC) curve, which is derived from the free entry condition, equates expected gain and cost of a vacancy

$$JC : \quad (1 - \omega)\left(\frac{(x^e - \hat{x})(1 - \tau)}{\pi^n + r} - F + H\right) - \frac{c}{q^f} = 0. \quad (1.17)$$

The first term is the expected gain of job creation for a firm, i.e. the firm’s after-tax share of excess output discounted by $\pi^n + r$. The gain is additionally raised or lowered depending on whether the hiring subsidy $H$ exceeds the firing tax $F$, or vice versa. The second term reflects the expected costs of job creation, i.e. the net flow cost $c$ times the average duration of a vacancy $1/q^f$.

$$JD : \quad (1 - \tau)\hat{x} + D + \frac{\pi^n(1 - \tau)}{\pi^n + r} \int_{\hat{x}}^{\infty} (\hat{x} - \tilde{x}) dG(\tilde{x}) - \frac{z}{1 - t} + rF - \frac{\omega}{1 - \omega}c\theta = 0. \quad (1.18)$$

The first line of the job destruction (JD) condition, which represents the ‘inside’ cut-off condition, gives the lowest acceptable joint inside value of a job, i.e. the

$^{12}$See appendix section 1.C for a detailed derivation of (1.17) to (1.20).
after-tax reservation product plus a wage subsidy $D$ and the option value of keeping a worker as her productivity might change. The second line can be interpreted as the joint outside value, which increases in $z$ and $\theta$, as both raise the worker’s outside option, and decreases in $F$. The analytic relationship of the ‘outside’ to the ‘inside’ productivity cut-off is novel compared to other studies that do not take endogenous job acceptance into account, i.e.

$$\bar{x} = \hat{x} + \frac{(\pi^n + r)}{(1 - \tau)} (F - H). \quad (1.19)$$

Observe that both cut-offs coincide in a policy free environment where $F = H = 0$. A hiring subsidy $H$ will put a wedge between those cut-offs in a way that agents more easily accept than destroy a job ($\bar{x} < \hat{x}$). A firing tax $F$ has the opposite consequence, $\bar{x} > \hat{x}$. Having derived all three decision variables $\theta$, $\hat{x}$, and $\bar{x}$, I can compute unemployment $u$. Just insert in the typical Beveridge curve (1.20), which is derived by setting the change in $u$, i.e. $\dot{u} = (L - u)\pi^nG(\bar{x}) - uq^w$, to zero, i.e.

$$u = \frac{\pi^nG(\bar{x})}{\pi^nG(\bar{x}) + q^w} \cdot L. \quad (1.20)$$

As mentioned, the recursion of the system reduces the problem to solving only two equations simultaneously. Therefore, I can conveniently analyze comparative statics in the JD-JC diagram$^{13}$, drawn in the $\theta$-$\hat{x}$-space (see Pissarides, 2000). The JC-curve is sloping downward because firms post fewer vacancies the higher $\hat{x}$, as average duration of a job decreases in $\hat{x}$. The JD-curve slopes upward because workers want to terminate jobs more easily the higher $\theta$, as their outside options increase in labor market tightness. Hence, the curves intersect at most once, as illustrated by figure 1.1, which makes the equilibrium unique in case of existence. I will now shortly address the effects of uncompensated changes in my policy instruments$^{14}$. A wage subsidy $D$ has no effect on the JC-curve but shifts out the JD-curve. Hence, equilibrium labor market tightness $\theta$ will go up, the reservation productivities $\bar{x} = \hat{x}$ will fall, leading to more job creation, more acceptance and less destruction. Therefore, unemployment will unambiguously decrease. A hiring subsidy $H$ works quite differently. While there is no effect on the JD-curve, the JC-curve will shift outward. This raises labor market tightness and consequently job creation as well as job destruction. Relative to job destruction, job acceptance is boosted,

$^{13}$See appendix section 1.F for more details.
$^{14}$See appendix section 1.F for the analytic derivation.
i.e. $\hat{x} < \bar{x}$. Whether job acceptance rises or falls in absolute terms is ambiguous. Proposition 1.2.1 states a condition for the direction of the absolute effect.

**Proposition 1.2.1.** Hiring subsidy and job acceptance: A hiring subsidy can lead to more or less job acceptance. Assume for simplicity that $t = \tau = 0$. Whenever $\nabla^{-1} \omega c(1 - \Psi) < \pi^n + r$ the effect of $H$ on $\bar{x}$ will be negative, leading to more job acceptance.

Proof. Differentiating (1.19) w.r.t. $H$ gives $\frac{\partial \hat{x}}{\partial H} = \frac{\partial \hat{x}}{\partial H} - (\pi^n + r)$. Inserting for $\frac{\partial \hat{x}}{\partial H}$ derived using the implicit function theorem and rearranging completes the proof. \[15\]

**Figure 1.1:** Uncompensated comparative statics

(a) The job destruction, job creation diagram

(b) Equilibrium effects

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*This column gives the effect on $\bar{x}$ in relation to $\hat{x}$.\[16\]

A firing tax $F$ has very similar but inverted effects compared to a hiring subsidy $H$. While the JC-curve moves inward to the same extent, additionally also the JD-curve shifts outward as long as $r > 0$. Hence, if $F = H$ rise simultaneously all shifts of the JC-curve cancel, while the shift in the JD-curve remains.

**Proposition 1.2.2.** The $F = H$ scheme: Let $r > 0$. Then $F = H > 0$ leads to more job creation and acceptance, less job destruction and consequently reduced unemployment.

\[15\] $\nabla$ denotes the determinant of the JD-JC system which is always positive. $\Psi$ is the derivative of the conditional expectation with respect to the cut-off. See appendix section 1.F for details.

\[16\] The sufficient and necessary condition for $\hat{x}$ to increase (assuming $F = H$ and $\omega = \eta$ for simplicity) is: $[\pi^n + r + q^n G(\hat{x})] \bar{x} > [q(\theta) - \pi^n] (r + \pi^n) \Gamma$. Hence, it is also sufficient to raise $u$.\[16\]
The positive effect of this scheme, also described in Ligthart and Heijdra (2000) and Heijdra and Ligthart (2002), can be explained as follows. Looking at the life cycle of a job, a \( F = H \) scheme can be compared to an interest free loan to the firm, as it gets \( H \) at the beginning of a job and eventually pays back the same amount without capital user costs. The gain is therefore reflected in the \( rF \)-term in the job destruction condition (1.18). Due to the dynamic structure of the model the alternative interpretation when considering the cross-section of firms at a specific point in time is that \( F = H > 0 \) implies redistribution from the firing to the hiring firms.

A recruitment subsidy affects both curves. Both shift outward leading to an increase in labor market tightness, but the JC-curve moves stronger which implies more job destruction. Compared to a hiring subsidy which is only paid if a job is created, a recruitment subsidy is received by the firm irrespective of whether a match occurs or not. The main consequence is that a hiring subsidy will partly go to the worker, while the latter subsidy is already sunk in the wage bargaining. All effects are summarized in table 1.1. These uncompensated comparative static exercises provide intuition through which channels my policy instruments work. In order to characterize optimal policy I have to develop a notion of efficiency and I have to close the government’s budget constraint to restrict the analysis to policy that is implementable. This is done in the next section.

1.2.2 Efficiency and the optimal policy mix

Efficiency can be distorted in many ways. I will consider two possibilities: first, the typical search externality that comes from the way workers and firms are matched. Second and more at the focus my analysis, I will consider a firing externality in the spirit of BT08 stemming from the requirement to finance unemployment benefits which is not taken into account by the agents. I will concentrate on this fiscal externality and not on the problem of how unemployment compensation, which I take as given, should be optimally set. Michau (2011) explicitly models the insurance problem with risk-averse workers in a comparable setup and finds that the welfare optimum requires full insurance, i.e. \( w = z \) and output maximization. As full insurance is incompatible with Nash bargained wages, I condition my efficiency analysis on the prior implementation of a partial insurance system. I assume that insurance is not perfect
but that $b$ is set such that the value of non-work $z$ is close to the value of work\textsuperscript{17}. My model with risk-neutrality can then be considered as a linear approximation of a more complex model that features concavity in the utility function (see Hagedorn and Manovskii, 2008 for a similar argumentation). The quality of the approximation naturally decreases in the difference of $w$ and $z$, which as argued above, is assumed to be small. In order to analyze named inefficiencies I compute the solution to the social planner’s problem of maximizing total output\textsuperscript{18}, which is given by the following three reduced equations for socially optimal job creation, job destruction, and job acceptance:\textsuperscript{19}

\begin{align}
(1 - \eta) \frac{x^e - \hat{x}}{\pi^n + r} - \frac{C}{q^f} &= 0, \\
\hat{x} + \frac{\pi^n}{\pi^n + r} \int_{\hat{x}}^{\infty} (\tilde{x} - \hat{x}) dG(\tilde{x}) - h - \frac{\eta}{1 - \eta} C \theta &= 0, \\
\bar{x} &= \hat{x}.
\end{align}

Comparing those relations with the decentralized equilibrium equations (1.17) to (1.19) in a policy free world, i.e. $b = F = \tau = t = D = H = R = 0$, reveals that they coincide if and only if $\omega = \eta$ (Hosios, 1990). From now on I will follow a Ramsey approach and assume that unemployment compensation $b > 0$ is exogenously given and has to be financed with the least possible distortions using my instruments. Subtracting (1.21) to (1.23) from (1.17) to (1.19) gives the conditions that the policies in question have to fulfill to restore efficiency,

\begin{align}
\frac{x^e - \hat{x}}{\pi^n + r} \left[ (1 - \omega)(1 - \tau) - (1 - \eta) \right] + \frac{R}{q^f} &= (1 - \omega)(H - F), \\
-\tau (\hat{x} + \pi^n \Gamma) - \frac{b + \eta h}{1 - t} + D + rF - C \theta \left[ \frac{\omega}{1 - \omega} - \frac{\eta}{1 - \eta} \right] + \frac{\omega}{1 - \omega} R \theta &= 0, \\
F &= H.
\end{align}

\textsuperscript{17}This is also reflected in the calibration choices later on.
\textsuperscript{18}In case of risk-neutral agents the solutions to the problems of maximizing output or utilitarian welfare coincide.
\textsuperscript{19}See appendix section 1.D for derivation.
where \( \Gamma \equiv \frac{1}{r+\pi} \int_{\hat{x}}^\infty (\hat{x} - \tilde{x}) \ dG(\tilde{x}) \). In addition, the government’s budget constraint\(^{20}\) must be met

\[
0 = (L - u)\bar{\omega}t + (L - u)\bar{x}\tau + q^w u (F - H) - \theta u R - (L - u)D - ub. \tag{1.27}
\]

The important consequence from introducing a job acceptance margin is that \( F = H \) has to hold even if the Hosios condition is not fulfilled\(^{21}\). In what follows I characterize two alternative implementations of the optimal allocation, one involving hiring and the other using wage subsidies. I depict the limitations to both schemes.

Let me first assume that the search externalities do not distort the equilibrium, i.e. \( \omega = \eta \). Inserting (1.26) in (1.24) reveals that output taxation and recruiting cost subsidization are not required for efficiency, hence \( \tau = R = 0 \). Unemployment benefits then have to be financed using the payroll tax \( t = \frac{b}{\omega} \frac{u}{L-u} > 0 \), which is chosen to fulfill (1.27). As a compensated firing tax, \( F = H \), is budget neutral, I can set \( F \) in order to fulfill (1.25), hence \( F = \frac{b + \theta h}{(1-t)r} > 0 \).

**Proposition 1.2.3. Implementation 1a:** In case of unemployment compensation \( b > 0 \) and \( \omega = \eta \) it is possible to implement the socially optimal allocation and balance the budget using a payroll tax, \( t > 0 \), a firing tax and a hiring subsidy, \( F = H > 0 \).

Observe the difference compared to BT08. In their framework the optimal policy consists of zero payroll taxes and a firing tax to finance unemployment benefits and offset the involved distortions. Here, a firing tax will distort the acceptance margin unless a firing tax is fully compensated by a hiring subsidy. As both instruments together are budget neutral a firing tax cannot be used for financing unemployment compensation. Instead of the redistribution from the firms to the workers as in BT08, I require redistribution from employed to unemployed workers and from firing to hiring firms.

Now consider the case where \( \omega \neq \eta \). Observe that at least one of the two policy instruments \( \tau \) or \( R \), is needed to satisfy equation (1.24). First I focus on output taxation, hence setting \( R = 0 \). The efficient output tax rate\(^{22}\) is then given by

\(^{20}\)Note that in equilibrium the number of outflows \( \pi^u G(\chi)(L - u) \) is equal to the inflows \( q^u u \). Hence, \( F = H \) is budget neutral in equilibrium. One should keep in mind that the introduction of a \( F = H \) scheme shifts the JD-curve inward leading to more outflow out of and less inflow into unemployment. Hence, during transition the outlay on \( H \) will exceed the revenue generated by \( F \).

\(^{21}\)Because of lemma 1.2.1 this finding also generalizes to a framework with risk-averse workers and is independent of whether welfare or output is maximized.

\(^{22}\)Note that in a model with physical capital an output taxation would distort capital usage. This issue is ignored in this paper.
\( \tau = 1 - \frac{1-\eta}{1-\omega} \) which is smaller than zero i.e. a subsidy if \( \omega > \eta \) and positive if \( \omega < \eta \). Therefore, the budget-solving payroll tax rate will be higher (\( \omega > \eta \)) or smaller (\( \omega < \eta \)) compared to the benchmark tax rate where the Hosios condition holds. Again \( F \) is set to fulfill (1.25) and therefore the implementation of the optimal allocation is complete. Note that the case \( F < 0 \) cannot be ruled out now. Instead of \( \tau \) one could alternatively use \( R = \frac{x^r - \xi}{\pi^{a+1}}(\omega - \eta)q^f \) by the same argument.

Proposition 1.2.4. **Implementation 1b**: In case of unemployment compensation \( b > 0 \) and \( \omega \neq \eta \) it is possible to implement the optimal allocation and balance budget using a payroll tax \( t \), a firing tax and a hiring subsidy, \( F = H \), and at least one of the following two instruments: output (\( \tau \)) or recruitment (\( R \)) tax/subsidy.

MP03 do not explicitly consider the case of \( \omega \neq \eta \) but it is easy to see that their job creation curve can be moved to the optimum just by adjusting \( F \neq H \) accordingly. In my case this is not possible as \( F = H \) is always required to offset the distortions at the job acceptance margin. Hence, the job creation curve can only be shifted by additional instruments, such as an output or a recruitment tax/subsidy.

The above implementations might require the firing tax to be of considerable magnitude. This will certainly be an issue when firms are liquidity constrained, e.g. \( F \leq F_{\text{max}} \) (see BT08) which will eventually prevent the implementation of the optimal allocation. This becomes even more severe in the following extension. One can assume that \( F \) only partly improves the government’s budget, say by \( F_{\text{tax}} \) as a fixed part \( F_{\text{cost}} = F - F_{\text{tax}} \) reflects sunk firing costs, e.g. the administrative costs of a lay-off, etc. Obviously, \( F = H \) is no longer budget neutral, implying that the payroll tax \( t \) has to rise to close the budget constraint and \( F = H \) have to be even higher to undo the additional distortion of the increased payroll tax. Hence, it is more likely to hit \( F_{\text{max}} \).

Note that a wage subsidy \( D \) is not required for achieving efficiency but possibly provides an alternative implementation. For simplicity assume again that \( \omega = \eta \) and set \( F = H = \tau = R = 0 \). The lump-sum wage subsidy \( D \), in addition to unemployment compensation \( b \), is financed using a payroll tax \( t \), ergo \( D = \bar{w}t - \frac{u}{\ell - u}b \). The job destruction curve will coincide with its social optimal counterpart if and only if \( \frac{b}{1-t} + \frac{ht}{1-t} + \frac{u}{\ell - u}b = \bar{w}t \). For \( u > 0 \) I can derive a necessary condition for the replacement ratio, namely \( \frac{b}{\bar{w}} < \frac{1}{4} \). The contrapositive reads:
Proposition 1.2.5. Implementation 2: In case of unemployment compensation \( b > 0 \) and \( \omega = \eta \) it is not possible to implement the optimal allocation and balance budget using only a payroll tax \( t \) and a wage subsidy \( D \), if the replacement ratio is higher than 25%.

\[
\text{Proof. } \frac{b}{1-t} + \frac{ht}{1-t} + \frac{u}{L}\frac{b}{u} b = \omega t \overset{u>0}{\Rightarrow} \frac{b}{1-t} + \frac{ht}{1-t} < \omega t \Rightarrow \frac{b}{\omega} < (1-t) - \frac{th}{\omega} \Rightarrow \frac{b}{\omega} < \max_{t, \frac{h}{\omega} \in [0,1] \times [0,1)} \left[ t(1-t) - \frac{th}{\omega} \right] = \frac{1}{4} \iff \frac{b}{\omega} < \frac{1}{4}
\]

This implementation is very specific to the way I introduced those instruments, i.e. \( D \) being lump-sum and \( t \) being linear. A possible implementation with \( D > 0 \) and \( t > 0 \) would mimic a progressive tax schedule. As the condition of proposition 1.2.5 is hardly met in any OECD economy anyways I shall focus on implementation 1 in what follows.

1.3 An intergroup model with economic turbulence

So far, I focused on intragroup redistribution. Allowing for intergroup redistribution enriches the model considerably because it enables me to evaluate more realistic policies. MP03 find in a numerical simulation that a wage subsidy targeted at low-skilled workers and financed by high-skilled workers works quite well in bringing down overall unemployment\(^{23}\). Besides the connection via the government’s budget constraint, they assume the two skill classes to operate in complete juxtaposition. The issue that “targeting is likely to damage the quality and quantity of labor supply” (Bovenberg et al., 2000) is therefore hardly addressed. The aim of this section is to show how the optimal policy mix is altered by the presence of economic turbulence and I find that a scheme as proposed by MP03 might be considerably less effective in such an environment. The idea that increased economic turbulence affects labor market outcomes is related to Ljungqvist and Sargent (1998), who assume that unemployed workers lose their skills in the course of time\(^{24}\) as they cannot keep up to date with new production technologies. In a broader interpretation, these new production techniques and requirements emerge as a result of ongoing restructuring from manufacturing to services, spread of new information technologies, internationalization of production, etc. which all lead to expeditious

\(^{23}\) For the ‘European calibration’ they find that a 20% wage subsidy decreases low-skilled unemployment from 16.2% to 7.6% while the unemployment rate of high-skilled workers rises from 4.5% to only 4.9%.

\(^{24}\) Empirical evidence for skill loss upon separation or during unemployment, which is often approximated by the difference between the old wage and the re-employment wage, is widely documented. See for example Fallick (1996) for the U.S. and Burda and Mertens (2001) for Germany.
changes in the economic environment, and render previous ways of production obsolete. Hence, a worker who is only familiar with outdated techniques is less productive when confronted with state-of-the-art production technology.

The key differences compared to the simple intragroup model described above follow from the introduction of a second skill class with the property that the productivity distribution function of the high-skilled \((h)\) first-order stochastically dominates the cumulative distribution function (cdf) of the low-skilled \((l)\), i.e. \(G_h(x) \leq G_l(x), \forall x.\) Introducing economic turbulence is modeled as follows. High-skilled workers lose their skills conditional on job loss and during unemployment with probability \(\pi^l\), which means that they can only draw from \(G_l(\cdot)\) when they are matched again. Low-skilled workers, on the other hand, receive a skill upgrade during employment, reflecting ‘learning-on-the-job’, with probability \(\pi^h\), which allows them to draw a new productivity from \(G_h(\cdot)\) instead of \(G_l(\cdot)\). Hence, the skill composition is endogenous. For simplicity I assume that the skill of a specific worker can be observed by firms and the government at any time. Hence, a firm can direct search towards the skill class which is more profitable for the firm.\(^{25}\) An individual can be in four different states, employed with high or low skills and unemployed with high or low skills, where I assume that total labor force is normalized to 1, hence: \(e_l + u_l + e_h + u_h = L_l + L_h = 1.\) Transitions between these states are illustrated by figure 1.2 and are formally reported in appendix section 1.A. Note that I now additionally allow for exogenous, productivity-unrelated, separation at a rate \(\pi^x\), which does not provide additional analytic insight, but is important to quantitatively match the model to the data. Beside the productivity distributions I allow high- and low-skilled

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\(^{25}\)I abstract from undirected search of the form that firms cannot ex-ante distinguish workers by their skill and have to source from a single pool, as presented e.g. in Albrecht and Vroman (2002), as it would add an additional externality to my framework. Firms would not internalize the positive effect of employment on the average quality of the pool of workers from which they source, which would lead to inefficiently low job creation.
workers to differ in other dimensions, like the matching technologies, as well. Differences are indicated by the subscript $j \in \{h, l\}$. All the assumptions of the intragroup model still apply unless stated otherwise. Hence, the models are nested, i.e. the intragroup model is a special case of the intergroup model with $\pi^h = \pi^l = \pi^x = 0$ and dropped skill indices. The asset value of unemployment for the low-skilled workers is the same as before, while high-skilled workers lose $U_h$ in case they are not matched with probability $\pi^l$ and only get $U_l$ instead:

$$rU_j = z_j + q^w_j \left( W^e_j - U_j \right) + \mathbb{I}_h(j)(1 - q^w_h)\pi^l (U_l - U_h). \quad (1.28)$$

The value of working differs for both skill classes as follows. While the outside option of a low-skilled worker is only $U_l$, the possibility of a skill loss has to be incorporated in the outside option of a high-skilled worker, hence:

$$\bar{U} \equiv \pi^l U_l + (1 - \pi^l) U_h. \quad (1.29)$$

I turn to the firms’ side. As the skill of the workers can be perfectly observed, firms are able to discriminate and specifically post a vacancy for high- or low-skilled workers. A firm will enter the labor market that generates higher returns. I further assume that it can reassess this decision every period. Let me therefore define $V^m \equiv \max \{V_h, V_l\}$. The values of posting vacancies in the high- and the low-skill market, respectively, are given as

$$rV_j = -c_j + q^f_j \left( f^e_j + H_j - V_j \right) + (1 - q^f_j) (V^m - V_j), \quad \text{with } c_j \equiv C_j - R_j. \quad (1.30)$$

Employing a high-skilled worker yields a per-period return of $rJ_h$ similar to

---

26 $\mathbb{I}$ denotes the indicator function of form $\mathbb{I}_i(j) = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{if } j \neq i \end{cases}$.

27 Note that the ‘inside’ asset values ($\hat{W}_j(x)$ and $\hat{J}_j(x)$) are set up analogously and are not reported in the text but only in the appendix section 1.B for the sake of completeness.
before, while $r_j I_l$ again accounts for the possibility of a skill upgrade
\begin{align*}
    r_j I_j(x) &= (1 - \tau_j) x - w_j(x) + D_j + \pi^\ell \left[ (V^m - F_j) - I_j(x) \right] \\
    &+ \pi^n \left[ (1 - G_j(\hat{x}_j)) \hat{f}^\ell_j + G_j(\hat{x}_j)(V^m - F_j) - I_j(x) \right] \\
    &+ I_l(j) \pi^h \left[ (1 - G_h(\hat{x}_h)) \hat{f}^h_j + G_h(\hat{x}_h)(V^m - F_h) - I_l(x) \right]. \tag{1.31}
\end{align*}

Wages are again determined by Nash bargaining and are related in the following way:
\[ w_j(x) = \hat{w}_j(x) - r_j \omega (F_j - H_j), \]
where \( r_h = r + \pi^x + \pi^n \) and \( r_l = r_h + \pi^h \). Wages now do not only depend on their ‘own’ endogenous variables and parameters but also on those of the other skill group\(^{28}\). Importantly, this dependence is asymmetric. While wages of both skill classes increase in the own outside options, \( \partial w_h / \partial U_h > 0 \) and \( \partial w_l / \partial U_l > 0 \), as before, this is not true for the ‘cross terms’, \( \partial w_h / \partial U_l < 0 \) and \( \partial w_l / \partial U_h < 0 \). These derivatives capture policy spill-over of a targeting scheme. Subsidizing low-skilled workers will increase \( U_l \) and lead high-skilled workers to bargain a higher wage as their fallback option, which includes that they eventually become low-skilled, increases\(^{29}\). By contrast, if high-skilled workers are subsidized, low-skilled workers will bargain a lower wage because working with low skills includes the increased option value of becoming high-skilled. I will now characterize the equilibrium.

### 1.3.1 Equilibrium

The equilibrium vector \( \langle u_h, u_l, e_h, e_l, \theta_h, \theta_l, x_h, x_l, \hat{x}_h, \hat{x}_l \rangle \) is pinned down by equations (1.32) to (1.34) and the steady state flow equations (1.39). In comparison to the intragroup model, the job creation conditions hardly change
\begin{align*}
    JC_j : \quad (1 - \omega) \left( \frac{(x_j^e - \hat{x}_j)(1 - \tau_j)}{r_j} - F_j + H_j \right) &= \frac{c_j}{q_j}. \tag{1.32}
\end{align*}

The job destruction conditions are now more involved. After defining \( \Gamma_j \equiv \frac{1}{r_j} \int_{\hat{x}_j}^{\infty} (\hat{x} - \hat{x}_j) dG_j(\hat{x}) \), they read
\begin{align*}
    JD_j : \quad & (1 - \tau_j) \hat{x}_j - \hat{w}_j(\hat{x}_j) + D_j + rF_j - I_l(j) \pi^h (F_h - F_l) \\
    &+ (1 - \omega) \pi^n (1 - \tau_j) \Gamma_j + I_l(j)(1 - \omega) \pi^h (1 - \tau_h) \Gamma_h = 0. \tag{1.33}
\end{align*}

\(^{28}\)See appendix section 1.C for an explicit derivation of all four wage schedules.

\(^{29}\)Note that this effect should be even stronger if workers are risk-averse.
The relationship of the cut-off productivities, representing the job acceptance decision, is given by

\[ JA_j : \quad x_j = \hat{x}_j + \frac{r_j}{1 - \tau_j} (F_j - H_j). \quad (1.34) \]

Analogously to before equilibrium is partly recursive. After inserting (1.34) in (1.32) in order to compute \( x^e_j \), one can solve the remaining JD-JC system of four equations for \( \theta_h, \theta_l, \hat{x}_h, \) and \( \hat{x}_l \). Knowing \( \theta_j, \hat{x}_j, \) and \( x_j \) allows to solve for \( e_j \) and \( u_j \) using (1.39).

### 1.3.2 Efficiency and the optimal policy mix

As before I start out by computing the solution to the social planner’s problem, which is documented in appendix section 1.E. Again, efficiency in a policy-free world is guaranteed if and only if \( \omega = \eta \). Hence, the Hosios (1990)-condition generalizes to the complex intergroup model. I use the same Ramsey approach as before, i.e. \( b_h \) and \( b_l \) are exogenously given and have to be financed with the least possible distortion. As the implementation of the optimum should be feasible I am bound to the following budget constraint that allows for intergroup redistribution

\[ 0 = GB_h + GB_l, \quad (1.35) \]

\[
GB_j = e_j [\bar{\omega} t_j + \bar{x}_j \tau_j - D_j] - u_j b_j - \theta_j u_j R_j - a_j^w u_j H_j \\
+ e_j \tau^x G_j(\hat{x}_j) F_j + e_j \tau^x F_j + \Pi_h(j) e_l \tau^h G_h(\hat{x}_h) F_h. \quad (1.36)
\]

Many insights from the intragroup model generalize to the extended model. First, \( F_j = H_j \) is again a necessary condition for efficiency. Second, if \( \omega = \eta \) holds I do not require output taxation \( \tau_j \) or a recruiting subsidy \( R_j \). If \( \omega \neq \eta \) I need at least one of those instruments. These are important guidelines for finding an implementation of the optimal allocation for the complex intergroup model which is a non-trivial task because of several complications. First, even \( b_j > 0 \) and \( b_i \neq j = 0 \) requires both payroll tax rates to be non-zero. Second, a \( F_j = H_j \)-scheme is only budget neutral if \( F_l = H_l \) as there are no taxes paid or subsidies received if a worker ‘upgrades’. Consequently, whenever \( F_l > H_l \) the effect on the budget is negative, if \( F_l < H_l \) it is positive. If I require \( F_h = F_l = H_h = H_l \) then the implementation of the efficient allocation is given by the vector \( \langle t_h, t_l, F_l \rangle \) that satisfies the government’s budget constraint (1.35) and pushes the JD-curves to their optimum, i.e. (1.62) minus (1.101) is equal to zero and (1.63) minus (1.100) is equal to zero. If I do not require \( F_l = H_l \) then I have an additional degree of freedom and can have many optimal imple-
1.3. AN INTERGROUP MODEL WITH ECONOMIC TURBULENCE

The central insight is that the idea of implementation 1 generalizes to the complex model. Hence, the $F = H$-scheme is robust to the presence of economic turbulence.

1.3.3 Simulation

Although the theoretical treatment gives a lot of insight I will perform some numerical simulations for two reasons. First, I provide quantitative evidence for the possible welfare gain when the optimal policy is implemented. Second, I will show that the quantitative relevance of the additional spill-over effects of targeting schemes, which arise in presence of economic turbulence, is considerable. I will do so by revisiting the effects of a cross-financed wage subsidy scheme proposed by MP03 who find that such a policy can considerably increase employment and output in their ‘European calibration’ case. For the sake of comparability I will focus on Germany as a representative European economy. The first task is to find a reasonable calibration for the model to fit German labor market characteristics. I specify the functional forms of $q_j(\cdot)$ and $G_j(\cdot)$ following MP03, Den Haan et al. (2005), or Ljungqvist and Sargent (1998, 2004)

$$q_j(\theta_j) = A_j \theta_j^{-\eta}, \quad (1.37)$$

and a uniform distribution on the interval $[\kappa, \bar{\kappa}]$

$$G_j(x) = \frac{x - \kappa_j}{\bar{\kappa}_j - \kappa_j}. \quad (1.38)$$

A period is chosen to be a month. Targeting an interest rate of 5% p.a. results in $r = 0.0041$. Nash bargaining is chosen to be symmetric as done by many authors. Estimates for the elasticity of the matching function vary between 0.45 (Fahr and Sunde, 2001) and 0.7 (Burda and Wyplosz, 1994). For simplicity, I abstract from inefficiencies generated by search externalities. Hence, I set $\eta = 0.5$ in order to fulfill the Hosios (1990)-condition. The expectations of the two productivity distributions were chosen to be $E(X_l) = 1$ and $E(X_h) = 1.35$. As data for per-worker productivity broken down into skill classes is not available, gross wages were used as proxies by assuming that productivities and wages are sufficiently proportional. During 2002 to 2006 a white collar worker earned approximately 1.35 times as much as a blue collar worker (Statistisches

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30 The simulations were performed using MATLAB. The code is available upon request.

31 See Hall and Milgrom (2008) for an additional motivation of setting $\omega = 0.5$. 

Bundesamt, 2007). Variances were set such that the model’s wage predictions result in a wage ratio of approximately 1 : 1.35. This implies \( \text{Var}(X_l) = \frac{1}{12} \) and \( \text{Var}(X_h) = \frac{1.3^2}{12} \). A crucial choice is the value of non-work \( z_j \) which was set as a compromise between two different calibration strategies. As mentioned earlier I impose linearity in the value of non-work, hence \( z_j = h + b_j \), which implies that the effects of \( db_j \) and \( dh \) are equal. I further assume that there is no skill specific difference in the value of home production. In line with the results of the OECD tax-benefit calculator I target replacement ratios of \( \frac{b_h}{w_h} = 0.6 \) and \( \frac{b_l}{w_l} = 0.65 \). Exploiting cross-country variation, Costain and Reiter (2008) estimate the semi-elasticity of unemployment with respect to the replacement ratio \( \frac{b}{w} \), i.e. \( \frac{d \ln u}{d \frac{b}{w}} \) approximately in the range of \([2, 3]\). I set a common \( h = 0.25 \) low enough to hit the upper bound of the Costain and Reiter (2008)-target, namely \( \frac{d \ln u}{d \frac{b}{w}} \approx 3.2^{32} \). This implies total replacement ratios of \( \frac{z_h}{w_h} = 0.77 \) and \( \frac{z_l}{w_l} = 0.87 \) and, in terms of productivity, \( \frac{z_h}{av.prodl_h} = 0.717 \) and \( \frac{z_l}{av.prodl_l} = 0.814 \). Those figures are just high enough to be in line with the corresponding value of 0.71 derived by Hall and Milgrom (2008) for the U.S. using a completely different calibration approach relying on estimates of the Frisch elasticity. As one would expect this number to be even higher in Germany, because of the more generous compensation system, 0.71 seems to be a reasonable choice as a lower bound. Hence, my calibration compromises between both targets. It addresses the argument of Hagedorn and Manovskii (2008) that the value of non-work is substantially high, but at the same time produces a realistic responsiveness of unemployment to changes in benefits. In order to finance the expenditure on \( b_h \) and \( b_l \) I set \( t_h = 0.065 \) and \( t_l = 0.05 \), which reflects progression in the existing tax system.

The transition probabilities \( \pi^l \) and \( \pi^h \) are chosen in order to replicate the empirical skill distribution. I use the following targets based on the publicly available statistics provided by the German federal employment agency (Bundesagentur für Arbeit, 2008). Among the unemployed the ratio of blue to white collar workers is approximately 60 %, hence \( \frac{u_l}{u} \approx 0.6 \). I further target \( \frac{e_l}{e} \approx 0.2 \), where the low-skilled are measured as workers with no professional education and apprentices. Given an unemployment rate of \( u = 0.1 \), this gives a skill composition of the labor force of \( \frac{L_L}{L} \approx 0.25 \). I set \( \pi^h = 0.01 \), which implies that

\[ \text{The model delivers } \frac{d \ln u}{d \pi^h} = 2.2, \frac{d b_h}{d \pi^h} = 0.6 \text{ and } \frac{d b_l}{d \pi^h} = 0.73. \]
it takes on average 8 years and 4 months to become high-skilled, conditional on no job loss. A skill loss occurs after 1 year and 10 months on average, i.e. $\pi^l = 0.05$. Those values are in line with the choices of Den Haan et al. (2005) and Ljungqvist and Sargent (2004). These papers and MP03 also inspire the choice for the rate at which new shocks arrive, i.e. $\pi^n = 0.02$. As I do not interpret the average duration of a vacancy or the number of vacancies but just target the duration of unemployment I am free to choose $C_h = 1.509$ and $C_l = 0.274$ in order to normalize $\theta_h = \theta_l = 1^{33}$. The probability of an exogenous split $\pi^x = 0.00668$ and the scaling factors $A_h = 0.563$ and $A_l = 0.148$ are set to replicate an unemployment rate of $u = 0.1$ and average duration of unemployment of 9 months (long term averages for 1998-2007, Bundesagentur für Arbeit, 2008). Table 1.1 summarizes the calibration choices and results for the decentralized economy, which serves as my benchmark.

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<td>$l$</td>
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<th>av. $w_j$</th>
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<th>tot. repl.</th>
<th>welfare</th>
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Note: ‘repl.’ gives the replacement ratio, i.e. $b_j/\bar{w}_j$. ‘tot. repl.’ is $\check{z}_j/\bar{w}_j$. ‘av.’ denotes average. ‘welfare’ is per-period in steady state. Other variables as in the paper.

In contrast, table 1.2 shows the results of the social optimum. The chosen welfare criterion increases by 5.6%. Unemployment is at 4% compared to 10%.

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33This normalization is more thoroughly described in Shimer (2005a).

34The chosen ratio of $A_h$ to $A_l$ is admittedly a little bit arbitrary but could be fixed if average duration of unemployment is known for each skill class separately. For the time being average duration of unemployment was set to 3 and a half months for high-skilled and a little bit more than 1 year for low-skilled.
while average duration of unemployment should optimally be 3 months instead of 9. Comparing the endogenous decision variables one can observe two things. First, reservation productivities for accepting and destructing jobs are inefficiently high, especially for the low-skilled who reject almost every second offer instead of one out of four which would be optimal. Second, job creation is inefficiently low. Again, this is more severe for low-skilled workers where market tightness is about one forth of what it should be.

### Table 1.2: Social planner’s solution

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#### Results

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Note: See table 1.1.

### Implementation of the optimal allocation

Let me now address possible implementations of the social optimum. We have learned from the previous sections that in case of \( \omega = \eta \) one does not require output taxation \( \tau_i \) or a recruiting subsidy \( R_i \). Further, given proposition 1.2.5 for the intragroup model and the high empirical replacement ratios an implementation relying on wage subsidies does not seem to be very promising. Hence, I try to implement the corresponding intergroup variant of the policy scheme suggested in proposition 1.2.3. I proceed as follows. First, I set \( F_j = H_j \). As \( \omega = \eta \) the job creation conditions coincide with their social optimal counterparts. Given \( F_j = H_j \) and \( b_j \) one can now compute the tax rates \( t_j \) that satisfy the two optimal job destruction conditions simultaneously. All the possible pairs of \( F_h = H_h \) and \( F_l = H_l \) that satisfy the budget constraint, i.e. set the budget surplus to 0, represent an implementation of the optimal allocation.
Figure 1.3 illustrates these socially optimal combinations. Moving along the

Figure 1.3: First best implementations in the intergroup model

(a) Budget surplus for efficient tax rates and all combinations of $H_j = F_j$

(b) Possible implementations of the optimal allocation

<table>
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<tr>
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<td>0.668</td>
</tr>
<tr>
<td>40</td>
<td>122.4</td>
<td>-0.033</td>
<td>0.594</td>
</tr>
<tr>
<td>60</td>
<td>125.6</td>
<td>-0.022</td>
<td>0.519</td>
</tr>
<tr>
<td>80</td>
<td>131.5</td>
<td>-0.014</td>
<td>0.443</td>
</tr>
<tr>
<td>100</td>
<td>139.2</td>
<td>-0.007</td>
<td>0.365</td>
</tr>
<tr>
<td>120</td>
<td>148.3</td>
<td>-0.001</td>
<td>0.287</td>
</tr>
<tr>
<td>140</td>
<td>158.3</td>
<td>0.004</td>
<td>0.208</td>
</tr>
<tr>
<td>160</td>
<td>169.0</td>
<td>0.008</td>
<td>0.129</td>
</tr>
<tr>
<td>180</td>
<td>180.3</td>
<td>0.012</td>
<td>0.050</td>
</tr>
<tr>
<td>200</td>
<td>192.0</td>
<td>0.016</td>
<td>-0.029</td>
</tr>
</tbody>
</table>

optimal isoline does not only change the combination of $F_h = H_h$ and $F_l = H_l$ but also the corresponding optimal tax rates as shown in table 1.3. The higher $F_h = H_h$ the higher $t_h$ has to be compared to $t_l$. The striking result is that such schemes involve tremendously high firing taxes and hiring subsidies. To get a feeling for magnitude: the lowest possible value for $F_l = H_l$ is still more than 100 times larger than the monthly wage of a low-skilled in the benchmark case. Hence, it is of interest how close one can get to the optimum if one is limited in the extend to which firing taxes and hiring subsidies can be introduced as the effects presumably work in a concave way. Consider the uniform policy case $F_j = H_j = 90$ representing a ‘half-way’ policy mix. Payroll taxes are adjusted to satisfy the budget constraint and keep the ratio $\frac{t_h}{t_l}$ constant. Such a policy gives a welfare gain of 4.4% instead of 5.6%. Unemployment is reduced to 5.7% and unemployment duration to 4.5 months. If the scheme is set to a value of $F_j = H_j = 10$, which amounts to approximately 7 months of average wage, unemployment is still reduced by 1%-point, unemployment duration by one month, while welfare increases by 1%. Another finding is that the considered instruments allow to trade off welfare and employment. Consider, for example, again $F_h = F_l = 10$, this time as only source of tax income and instead of using hiring subsidies, tax revenue is spent on recruitment subsidies $R_h = R_l = 0.24$. Such a scheme reduces overall unemployment to 6.3% and average duration of unemployment to 5.7 months, while it distorts job acceptance and creation and leaves welfare practically unchanged compared to the benchmark.
Cross-financed wage subsidy schemes

I argued in the theoretical part of this section how the presence of economic turbulence can create additional spill-over effects from targeted to untargeted workers. In this section I try to quantify this for a particular targeting scheme, namely a wage subsidy for low-skilled workers financed by high-skilled workers as studied by MP03. Although a policy like that does not fulfill the criteria of being optimal, MP03 propose it as a 'better than nothing' scheme especially useful to reduce unemployment. I show that this conclusion is overthrown when economic turbulence is taken into account. To have a reference point I first replicate the MP03 result in my model when turbulence is switched off, i.e. $\pi^h = \pi^l = 0$. Hence, the skill composition of the labor force is not endogenous anymore but exogenously fixed, i.e. $L_h = 0.7398$. To replicate my targets for unemployment, its duration and composition I have to recalibrate some of the remaining transition probabilities\(^{35}\). I then rerun the MP03 experiment by increasing $D_l$ stepwise from 0 to 0.5. This is done in an uncompensated way and also if financed by the high-skilled workers. Table 1.3 summarizes the results.

### Table 1.3: Effect of a low wage subsidy on unemployment rates

<table>
<thead>
<tr>
<th></th>
<th>$D_l$ change uncompensated</th>
<th>$D_l$ change compensated by $t_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no turbulence</td>
<td>turbulence</td>
</tr>
<tr>
<td>$D_l$</td>
<td>$u_h$</td>
<td>$u_l$</td>
</tr>
<tr>
<td>0.0</td>
<td>5.08</td>
<td>23.98</td>
</tr>
<tr>
<td>0.1</td>
<td>5.08</td>
<td>19.29</td>
</tr>
<tr>
<td>0.2</td>
<td>5.08</td>
<td>16.18</td>
</tr>
<tr>
<td>0.3</td>
<td>5.08</td>
<td>13.95</td>
</tr>
<tr>
<td>0.4</td>
<td>5.08</td>
<td>10.99</td>
</tr>
<tr>
<td>0.5</td>
<td>5.08</td>
<td>10.99</td>
</tr>
</tbody>
</table>

*Note: Unemployment rates are computed in percent relative to $L_j$. '-' denotes break down of equilibrium.*

As in MP03 a low-wage subsidy scheme seems to be very effective in reducing overall unemployment, which can be brought down to 7.02 % for $D_l = 0.66$ in the tax-compensated scenario. However, when I take economic turbulence into account, the results reverse. It is striking that even in the uncompensated case, i.e. the subsidy is given away for free, total unemployment will increase with $D_l$. Two effects, one boosting $u_h$ and the other dampening the reduction in $u_l$ come into play. In a first direct effect a rise in $D_l$ increases the value of working as low-skilled ($W_l$) and consequently the value of being unemployed ($U_l$) with low skills. That is where the mechanism stops in the non-turbulence

\(^{35}\)In detail, $A_h = 2.23$, $A_l = 0.28$, $\pi^u = 0.01$, $\pi^x_h = 0.0065$, and $\pi^x_l = 0.0185$. In addition, as wages slightly differ I have to set the tax rates (keeping the relative ratio constant) to $t_h = 0.054$ and $t_l = 0.046$. Again, I choose $C_h = 0.571$ and $C_l = 0.178$ in order to normalize $\theta_h = \theta_l = 1$. 
1.4 Conclusion

A dynamic model of equilibrium unemployment and bilateral wage bargaining is used to characterize optimal labor market policy in a possibly turbulent environment. The pre-policy equilibrium is distorted by a firing externality, created by an existing potentially unemployment insurance system, along three decision margins: job creation, job acceptance, and job destruction. I apply a Ramsey approach and try to find a solution to the problem of financing exogenously fixed unemployment benefits with the least possible distortions using a rich set of policy instruments: payroll, output, and firing taxes as well as wage, hiring, and recruitment subsidies. It is shown that the optimal policy mix consists of a payroll tax to finance unemployment compensation and a firing tax that is offset by a hiring subsidy. The latter part can be interpreted as redistribution from firing to hiring firms and helps to undo the distortions created by the payroll tax system. The reason is that a ‘firing tax equal hiring subsidy’-scheme, while not distorting job acceptance and job creation, leads to less job destruction as such a policy represents an interest free loan to the firm. The derived optimal policy mix deviates from the static framework results of Blanchard and Tirole (2008) who argue that benefits should be completely financed through firing taxes. This idea does not completely transfer to my dynamic set-up. In any case a firing tax has to be compensated one-for-one by a hiring subsidy to prevent distortions along the job acceptance margin. Hence, in the case of ‘unbalanced’ search externalities that distort job creation, the failure of the Hosios condition cannot be corrected by a spread between the firing tax...
and the hiring subsidy. Instead either an output or a recruitment tax/subsidy have to be used in addition.

The important feature of the derived policy mix is that it is robust to the introduction of economic turbulence in the interpretation of Ljungqvist and Sargent (1998), i.e. skill loss during unemployment. This is crucial as a lot of existing policy advice is rendered considerably less effective in that case. I demonstrate this by reassessing a cross-financed wage subsidy scheme for low-skilled workers as, for example, suggested by Mortensen and Pissarides (2003). While they assume skill classes to operate in complete juxtaposition, except for the connection via the government’s budget constraint, possible skill loss during unemployment implies that high-skilled workers become pickier concerning their acceptance and continuation decision as their fall back option, including subsidized low-skill employment, increases. The skill composition deteriorates as a result of such a targeting scheme and the finding of Mortensen and Pissarides (2003) that unemployment can be considerably reduced is overthrown. In conclusion, the paper argues that instead of redistribution from firing firms to unemployed workers (Blanchard and Tirole, 2008) or from high- to low-skilled workers (Mortensen and Pissarides, 2003), a scheme involving redistribution from firing to hiring firms should be preferred.
Appendix

1.A Laws of motion

\[
\dot{u}_h = e_h [\pi^x + \pi^n G_h(\hat{x}_h)] (1 - \pi^l) + e_l \pi^h G_h(\hat{x}_h) (1 - \pi^l)
- u_h \left[ (1 - q_h^w) \pi^l + q_h^w \right],
\]
\[
\dot{e}_l = (1 - e_h - e_l - u_h) q_l^w - e_l \left[ \pi^x + \pi^n G_l(\hat{x}_l) + \pi^h \right],
\]
\[
\dot{e}_h = e_l \pi^h (1 - G_h(\hat{x}_h)) + u_h q_h^w - e_h \left[ \pi^x + \pi^n G_h(\hat{x}_h) \right],
\]
\[
y_h = -y_h (\pi^x + \pi^n) + e_h \pi^n G_h(\hat{x}_h) + e_l \pi^h G_h(\hat{x}_h)
+ u_h \theta_h q_h(\theta_h) \hat{G}_h(\hat{x}_h),
\]
\[
y_l = -y_l (\pi^x + \pi^n + \pi^h) + e_l \pi^n \tilde{G}_l(\hat{x}_l)
+ (1 - u_h - e_l - e_h) \theta_l q_l(\theta_l) \tilde{G}_l(\hat{x}_l),
\]

where the partial expectation is defined as \( \tilde{G}_j(x) = \int_x^\infty \tilde{x} dG_j(\tilde{x}) \). Equilibrium states are derived by setting the left hand sides to zero.

1.B Unreported value functions and Nash bargaining

Unreported value functions:

\[
r \tilde{W}_j(x) = (1 - t_j) \tilde{w}_j(x) + \pi^x \left[ I_l(j) U_l + I_h(j) U - \tilde{W}_j(x) \right]
+ \pi^n \left[ (1 - G_j(\hat{x}_j)) \tilde{W}_j^e + G_j(\hat{x}_j) (I_l(j) U_l + I_h(j) U) - \tilde{W}_j(x) \right],
\]
\[
r \tilde{f}_j(x) = (1 - \tau_j) x - \bar{w}_j(x) + D_j + \pi^x \left[ (V^m - F_j) - \tilde{f}_j(x) \right]
+ \pi^n \left[ (1 - G_j(\hat{x}_j)) \tilde{f}_j^e + G_j(\hat{x}_j) (V^m - F_j) - \tilde{f}_j(x) \right],
\]

Nash bargaining implies:

\[
W_h - \bar{U} = \frac{\omega(1 - t_h)}{1 - \omega} (J_h + H_h) \quad \text{and} \quad \hat{W}_h - \bar{U} = \frac{\omega(1 - t_h)}{1 - \omega} (\hat{f}_h + F_h),
\]
\[
W_l - U_l = \frac{\omega(1 - t_l)}{1 - \omega} (J_l + H_l) \quad \text{and} \quad \hat{W}_l - U_l = \frac{\omega(1 - t_l)}{1 - \omega} (\hat{f}_l + F_l),
\]
or

\[
W_h - \bar{U} = \bar{w}_h \hat{s}_h \quad \text{and} \quad \hat{W}_h - \bar{U} = \bar{w}_h \hat{s}_h,
\]
\[
W_l - U_l = \bar{w}_l \hat{s}_l \quad \text{and} \quad \hat{W}_l - U_l = \bar{w}_l \hat{s}_l,
\]
\[ J_j + H_j = \tilde{\omega} j s_j \quad \text{and} \quad \hat{J}_j + F_j = \tilde{\omega} j \hat{s}_j, \quad (1.46) \]

where

\[ \tilde{\omega}_j \equiv \frac{\omega(1 - t_j)}{1 - \omega t_j} \quad \text{and} \quad \tilde{\omega}_j \equiv \frac{1 - \omega}{1 - \omega t_j} \quad \text{and} \quad (1 - \omega)\tilde{\omega}_j = \omega(1 - t_j)\tilde{\omega}_j. \]

**1.C Derivation of the equilibrium conditions**

This section formally derives the equilibrium conditions for the intergroup model. As the model is nested, the conditions for the simple intragroup model can be found by dropping the skill index and setting \( \pi^h = \pi^l = \pi^x \). Let me first define \( \tilde{r} = r + \pi^l \), \( r_h = r + \pi^x + \pi^n \), and \( r_l = r_h + \pi^h \). Equilibrium is determined by the free entry conditions (1.47) and the cut-off conditions (1.48), i.e.

\[ V_l = V_h = 0 \Rightarrow V^m = 0 \Rightarrow J^e_h = \frac{c_h}{q_h} - H_h \quad \text{and} \quad J^e_l = \frac{c_l}{q_l} - H_l, \quad (1.47) \]

\[ \hat{J}(\hat{x}_j) + F_j = 0 \Rightarrow \hat{x}_j \quad \text{and} \quad J(x_j) + H_j = 0 \Rightarrow x_j. \quad (1.48) \]

Take conditional expectation of (1.43), insert in (1.28) and eliminate \( J^e_l \) using the free entry condition (1.47) to get

\[ rU_l = z_l + \frac{\omega(1 - t_l)}{1 - \omega} c_l \theta_l. \quad (1.49) \]

Proceeding analogously for \( U_h \) results in

\[ rU_h = z_h + \frac{\omega(1 - t_h)}{1 - \omega} c_h \theta_h - \pi^l (U_h - U_l). \quad (1.50) \]

I use 1.50 and 1.49 to solve for the difference in the values of unemployment

\[ U_h - U_l = \frac{z_h - z_l}{\tilde{r}} + \frac{\omega (1 - t_h) c_h \theta_h - (1 - t_l) c_l \theta_l}{\tilde{r}}. \quad (1.51) \]

**Wages**

To get the wage equations proceed as follows. Multiplying (1.42) by \( r \) and rearranging gives \( \omega(1 - t_h)r\hat{f}_h(x) - (1 - \omega)r\hat{W}_h(x) = -(1 - \omega)r\bar{U} - \omega(1 - t_h)rF_h \). Replace \( r\hat{W}_h(x) \) and \( r\hat{f}_h(x) \) by (1.40) and (1.41). Most of the remaining values cancel out after eliminating them using the first-order conditions from the Nash bargaining (1.44) to (1.46), and their conditional expectations. Solving for \( \hat{w}_h(x) \) gives

\[ \hat{w}_h(x) = \omega [(1 - \tau_h)x + D_h + rF_h] + \frac{1 - \omega}{1 - t_h} r\bar{U}. \quad (1.52) \]
Eliminating the remaining values of being unemployed, realizing that \( r\bar{U} = r(1 - \pi^l) [U_h - U_l] + rU_l \), results in

\[
\hat{\omega}_h(x) = \frac{1 - \omega}{1 - t_h} \left( \frac{1 - \tau_h}{\bar{r}} z_h + \frac{\pi^l (1 + r)}{\bar{r}} z_l \right) + \omega \left[ (1 - \tau_h) x + D_h + rF_h \right] + \omega \left[ \frac{(1 - \tau^l) r}{\bar{r}} c_h \theta_h + \frac{1 - t_l}{1 - t_h} \frac{\pi^l (1 + r)}{\bar{r}} c_l \theta_l \right].
\] (1.53)

The derivation of the ’outside’ wage works analogously and results in \( w_h(x) = \hat{\omega}_h(x) - r_h \omega(F_h - H_h) \) given

\[
w_h(x) = \omega \left[ (1 - \tau_h) x + D_h - (\pi^x + \pi^n) F_h + r_H H_h \right] + \frac{1 - \omega}{1 - t_h} r\bar{U}.
\] (1.54)

I proceed the same way to get \( \hat{\omega}_l(x) \) and \( w_l(x) \).

\[
\hat{\omega}_l(x) = \omega \left[ (1 - \tau_l) x + D_l - \pi^h (F_h - F_l) + rF_l \right] + \frac{1 - \omega}{1 - t_l} \left[ rU_l - \pi^h (1 - \pi^l)(U_h - U_l) \right] + \omega \pi^h (1 - G_h(\hat{x}_h)) \frac{t_h - t_l}{1 - t_l} \approx \hat{\omega}_h \Sigma.
\] (1.55)

Note that in case of \( t_l \neq t_h, \hat{\omega}_h \Sigma\) does not drop out and is replaced by \( \frac{\hat{\omega}_h}{q^l} + \hat{\omega}_h \Sigma \).

See below for the derivation. Eliminating the values of being unemployed gives

\[
\hat{\omega}_l(x) = -\frac{1 - \omega}{1 - t_l} \pi^h (1 - \pi^l) \left[ \frac{z_h - z_l}{\bar{r}} + \frac{\omega}{1 - \omega} \left( 1 - t_h \right) c_h \theta_h - \left( 1 - t_l \right) c_l \theta_l \right] + \omega \left[ (1 - \tau_l) x + D_l - \pi^h (F_h - F_l) + rF_l + c_l \theta_l \right] + \frac{1 - \omega}{1 - t_l} z_l + \omega \pi^h (1 - G_h(\hat{x}_h)) \frac{t_h - t_l}{1 - t_l} \left[ \frac{c_h}{q^l_h} + \hat{\omega}_h \Sigma \right].
\] (1.56)

Similar to before the ’outside’ wage is given by \( w_l(x) = \hat{\omega}_l(x) - r_l \omega(F_l - H_l) \) knowing that

\[
w_l(x) = \omega \left[ (1 - \tau_l) x + D_l - \pi^h F_h - (\pi^x + \pi^n) F_l + r_l H_l \right] + \frac{1 - \omega}{1 - t_l} \left[ rU_l - \pi^h (1 - \pi^l)(U_h - U_l) \right] + \omega \pi^h (1 - G_h(\hat{x}_h)) \frac{t_h - t_l}{1 - t_l} \approx \hat{\omega}_h \Sigma.
\] (1.57)
For the derivation of $\Sigma$ I start out by noting that the surplus functions are linear of form $S_h(x) = S^0_h + S^1_h x$, and further

$$\hat{S}_h(x) = S_h(x) + (1 - \omega t_h) (F_h - H_h) = S^0_h + S^1_h x + (1 - \omega t_h) (F_h - H_h), \quad (1.58)$$

as will be established below. Taking conditional expectation gives

$$\hat{S}^\xi_h = \int_{\hat{x}_h}^{\infty} \left[ \frac{S^0_h + S^1_h \hat{x} + (1 - \omega t_h) (F_h - H_h)}{1 - G_h(\hat{x})} \right] dG_h(\hat{x})$$

$$= S^0_h + (1 - \omega t_h) (F_h - H_h) + S^1_h \frac{\tilde{G}(\hat{x})}{1 - G_h(\hat{x})}. \quad (1.59)$$

Taking conditional expectation of $S_h(x) = S^0_h + S^1_h x$, eliminating $S^0_h$ by using (1.59) and inserting for $S^1_h$ establishes $\hat{S}^\xi_h = S^\xi_h + \Sigma$. Combine (1.47) and (1.46) to get $S^\xi_h = \frac{c_h}{q_h} \frac{1}{\omega h}$ which gives $\omega_h \hat{S}^\xi_h = \frac{c_h}{q_h} + \omega_h \Sigma$, with

$$\Sigma = \frac{(1 - \tau_h)(1 - \omega t_h)}{r_h S^1_h} \left[ \frac{\tilde{G}(\hat{x}_h)}{1 - G_h(\hat{x}_h)} - \frac{\tilde{G}(x_h)}{1 - G_h(x_h)} \right] + \frac{(1 - \omega t_h)}{S_h(x)} (F_h - H_h).$$

Note that $\Sigma = 0$ if $F_h = H_h$ because it also implies $x_h = \hat{x}_h$ as I will prove below.

**Job creation conditions**

The job creation conditions are derived as follows. Subtract (1.41) evaluated at $\hat{x}_j$ from (1.31) and replace $\hat{f}_j(\hat{x})$ by $-F_j$ using (1.48). Taking conditional expectation w.r.t. $x_j$ and replacing $f^j_\tau$ using (1.47) gives the job creation curves

$$JC_j : (1 - \omega) \left( \frac{(x^j_f - \hat{x}_j)(1 - \tau_j)}{r_j} - F_j + H_j \right) = \frac{c_j}{q^j_f}. \quad (1.60)$$

**Job destruction conditions**

First define: $\Gamma_j \equiv \frac{1}{r_j} \int_{\hat{x}_j}^{\infty} (\hat{x} - \hat{x}_j) dG_j(\hat{x})$. Subtract (1.41) evaluated at $\hat{x}_j$ from themselves and eliminate $\hat{f}_j(\hat{x})$ by $-F_j$ again using (1.48). Use the conditional expectation w.r.t. $\hat{x}_j$ of the resulting expressions $\hat{f}_h(x)$ and $\hat{f}_i(x)$ to eliminate $\hat{f}_j$ in (1.41). Evaluate again at $\hat{x}_j$ and make use of (1.48) to arrive at

$$JD_j : (1 - \tau_j) \hat{x}_j - \omega_j(\hat{x}_j) + D_j + r F_j - I_l(j) \pi^h(F_h - F_i) + (1 - \omega) \pi^h(1 - \tau_j) \Gamma_j + I_l(j) (1 - \omega) \pi^h(1 - \tau_h) \Gamma_h = 0. \quad (1.61)$$

Eliminating the wages and diving by $(1 - \omega)$ then gives the final job destruction...
curves

\[
(1 - \tau_h) \hat{S}_h + D_h + rF_h - \frac{1}{1 - t_h} \left[ \frac{(1 - \pi^l)r}{\bar{r}} z_h + \frac{\pi^l(1 + r)}{\bar{r}} z_l \right] \\
- \frac{\omega}{1 - \omega} \left[ \frac{(1 - \pi^l)r}{\bar{r}} c_h \theta_h + \frac{1 - t_l}{1 - t_h} \frac{\pi^l(1 + r)}{\bar{r}} c_l \theta_l \right] + \pi^h (1 - \tau_h) \Gamma_h = 0,
\]

(1.62)

\[
(1 - \tau_l) \hat{S}_l + D_l + rF_l - \pi^h (F_h - F_l) - \frac{z_l}{1 - t_l} - \frac{\omega}{1 - \omega} c_l \theta_l \\
+ \frac{\pi^h (1 - \pi^l)}{1 - t_l} \frac{z_h - z_l}{\bar{r}} + \pi^h (1 - \pi^l) \frac{\omega}{1 - \omega} \left[ \frac{1 - t_h}{1 - t_l} \frac{c_h \theta_h}{\bar{r}} - \frac{c_l \theta_l}{\bar{r}} \right] \\
- \pi^h (1 - G_h (\hat{S}_h)) \frac{\omega}{1 - \omega} \frac{t_h - t_l}{1 - t_l} \left[ \frac{c_h}{q_h} + \tilde{\omega}_h \Sigma \right] \\
+ \pi^h (1 - \tau_l) \Gamma_l + \pi^h (1 - \tau_h) \Gamma_h = 0.
\]

(1.63)

Cut-off relationships

The relation between the reservation productivities $\bar{x}_j$ and $\hat{x}_j$ stems from a simple observation. The cut-off conditions in (1.48) in combination with (1.42) and (1.43) imply that firms and workers will always mutually agree on creating and destroying jobs. Hence, $x$ and $\hat{x}$ set the joint surpluses to 0. The surpluses in equilibrium are given by

\[
S_h(x) = W_h(x) + J_h(x) - \bar{U} + H_h \quad \text{and} \quad S_l(x) = W_l(x) + J_l(x) - U_l + H_l
\]

(1.64)

\[
\hat{S}_h(x) = \hat{W}_h(x) + \hat{J}_h(x) - \bar{U} + F_h \quad \text{and} \quad \hat{S}_l(x) = \hat{W}_l(x) + \hat{J}_l(x) - U_l + F_l
\]

(1.65)

Note that by lemma 1.2.1 both surplus functions and hence cut-offs coincide if $F_j = H_j$, a result which even holds in a more general framework with non-linear utility and non-linear payroll tax. Given my assumptions, observe that for the same $x$ the difference between the surplus functions is given by $S_j(x) - \hat{S}_j(x) = -\frac{t_j}{\bar{r}_j} (w_j(x) - \hat{w}_j(x)) + H_j - F_j = -(1 - \omega t_j) (F_j - H_j)$, which is independent of $x$. Hence, the surplus functions have the following linear structure

\[
S_j(x) = S^0_j + S^1_j x - (1 - \omega t_j) (F_j - H_j),
\]

(1.66)

\[
\hat{S}_j(x) = S^0_j + S^1_j x.
\]

(1.67)

From (1.29) and (1.31) I infer that $S^1_j = \frac{(1 - \tau_j)(1 - \omega t_j)}{t_j}$. The cut-offs solving
\( S_j(x) = 0 \) and \( S_j(\hat{x}) = 0 \) are therefore given by
\[
\begin{align*}
x_j &= -\frac{S_j^0 - (1 - \omega t_j)(F_j - H_j)}{S_j^1}, \\
\hat{x}_j &= -\frac{S_j^0}{S_j^1}.
\end{align*}
\]
Hence, the relationship of the cut-offs can be written as
\[
J_{A_j} : x_j = \hat{x}_j + \frac{r_j}{1 - \tau_j}(F_j - H_j).
\]

1.D Social planner’s optimum in the simple intragroup model

The constrained social optimum is derived by maximizing the social welfare function \( \Omega(\cdot) \) subject to the matching constraints and the evolution of total production \( y \), hence
\[
\max_{\{x, \hat{x}, \theta\}} \Omega = \max_{\{x, \hat{x}, \theta\}} \int_0^\infty e^{-rt}(y + uh - C\theta u)dt
\]
subject to
\[
\begin{align*}
\dot{u} &= \pi^n G(\hat{x})(L - u) - q^w u, \\
\dot{y} &= u\theta q(\theta) \int_{\hat{x}}^\infty \hat{x} dG(\hat{x}) + (L - u)\pi^n \int_{\hat{x}}^\infty \hat{x} dG(\hat{x}) - \pi^ny.
\end{align*}
\]
I set up the present-value Hamiltonian
\[
\mathcal{H} = e^{-rt}(y + uh - C\theta u) + \lambda_1 \left[\pi^n G(\hat{x})(L - u) - q^w u\right] + \lambda_2 \left[u\theta q(\theta) \int_{\hat{x}}^\infty \hat{x} dG(\hat{x}) + (L - u)\pi^n \int_{\hat{x}}^\infty \hat{x} dG(\hat{x}) - \pi^ny\right]
\]
The optimality conditions, i.e. \( \frac{\partial \mathcal{H}}{\partial x} = 0, \frac{\partial \mathcal{H}}{\partial \hat{x}} = 0, \frac{\partial \mathcal{H}}{\partial u} = 0, \frac{\partial \mathcal{H}}{\partial \theta} = -\dot{\lambda}_1, \frac{\partial \mathcal{H}}{\partial \pi^n} = -\dot{\lambda}_2 \), imply (1.75) to (1.79), i.e.
\[
\begin{align*}
\lambda_1 - \hat{x}\lambda_2 &= 0, \\
\lambda_1 - \hat{x}\lambda_2 &= 0.
\end{align*}
\]
From (1.75) and (1.76) one can infer that the cut-off productivities irrespective of whether one arrives at or has already been in a job coincide, i.e. \( \check{x} = \hat{x} \). From now on I will just use \( \check{x} \). Before stating the remaining first-order conditions
define 
\[
\tilde{G}(\hat{x}) \equiv \int_{\hat{x}}^{\infty} \tilde{x} \, dG(\tilde{x})
\]
and \(\Gamma \equiv \frac{1}{\pi + \pi'} \int_{\hat{x}}^{\infty} (\hat{x} - \tilde{x}) \, dG(\tilde{x}),\)

\[
-e^{-rt} C - \lambda_1 (1 - \eta) q(\theta) (1 - \tilde{G}(\tilde{x})) + \lambda_2 (1 - \eta) q(\theta) \tilde{G}(\tilde{x}) = 0, \tag{1.77}
\]

\[
-e^{-rt} (h - C\theta) - \lambda_1 [\pi^n G(\hat{x}) + \eta^{\omega}] + \lambda_2 [\theta q(\theta) - \pi^n] \tilde{G}(\tilde{x}) = -\dot{\lambda}_1, \tag{1.78}
\]

\[
e^{-rt} - \pi^n \lambda_2 = -\dot{\lambda}_2. \tag{1.79}
\]
Eliminating \(\lambda_1\) in (1.77) using (1.75) gives

\[
-e^{-rt} C + \lambda_2 (1 - \eta) q(\theta) (r + \pi^n) \Gamma = 0, \tag{1.80}
\]
which implies the following relationships for \(\lambda_1\) and \(\lambda_2\)

\[
\lambda_1 = \frac{e^{-rt}C\hat{x}}{(1 - \eta)q(\theta)(r + \pi^n)\Gamma} \quad \text{and} \quad \lambda_2 = \frac{e^{-rt}C}{(1 - \eta)q(\theta)(r + \pi^n)\Gamma}. \tag{1.81}
\]
Differentiating (1.80) w.r.t. \(t\) and subtracting (1.80) again results in the following relations

\[
\dot{\lambda}_2 = -\lambda_2 r \quad \text{and consequently} \quad \dot{\lambda}_1 = -\lambda_1 r. \tag{1.82}
\]
Inserting for \(\lambda_2\) and \(\dot{\lambda}_2\) in (1.79) and rearranging gives the reduced optimality condition that has a similar structure compared the job creation condition

\[
(1 - \eta) \frac{x^e - \hat{x}}{\pi^n + r} - \frac{C}{q^l} = 0. \tag{1.83}
\]
To derive the last reduced optimality condition, i.e. the job destruction condition counterpart, I eliminate \(\lambda_1, \lambda_2\) and \(\dot{\lambda}_1\) in (1.78) and rearrange

\[
\hat{x} - h + \pi^n \Gamma - \frac{\eta}{1 - \eta} C\theta = 0 \tag{1.84}
\]

1.E Social planner’s optimum in the intergroup model

Again I maximize discounted social welfare

\[
\int_{0}^{\infty} e^{-rt} \left[ y_h + y_l + (u_h + u_l)h - u_h C_h \theta_h - u_l C \theta_l \right] dt \tag{1.85}
\]
where \(u_l = (1 - u_h - e_h - e_l)\), subject to the evolution of the employment states \(\hat{u}_h, \hat{e}_l, \hat{e}_h\) and of total production \(\hat{y}_h\) and \(\hat{y}_l\) as given by (1.39), over the choice variables \(x_j, \hat{x}_j\) and \(\theta\). I set up the present-value Hamiltonian

\[
H = e^{-rt} \left[ y_h + y_l + (1 - e_h - e_l)h - u_h C_h \theta_h - (1 - u_h - e_h - e_l) C \theta_l \right] + \lambda_1 \hat{u}_h + \lambda_2 \hat{e}_l + \lambda_3 \hat{e}_h + \lambda_4 \hat{y}_h + \lambda_5 \hat{y}_l. \tag{1.86}
\]
The optimality conditions \( \frac{\partial H}{\partial x_j} = 0, \frac{\partial H}{\hat{x}_j} = 0 \) imply

\[
\lambda_1(1 - \pi^l) - \lambda_3 - \hat{x}_j \lambda_4 = 0 \quad \text{and} \quad \lambda_1(1 - \pi^l) - \lambda_3 - \hat{x}_j \lambda_4 = 0, \tag{1.87}
\]

\[
\lambda_2 + x_j \lambda_5 = 0 \quad \text{and} \quad \lambda_2 + \hat{x}_j \lambda_5 = 0. \tag{1.88}
\]

Hence, reservation productivities have to coincide again, i.e. \( x_j = \hat{x}_j \). For simplicity will just use \( x_j \) from now on. Define

\[
\tilde{G}_j(x_j) \equiv \int_{x_j}^{\infty} \tilde{x} \, dG_j(\tilde{x}) \quad \text{and} \quad \Gamma_j \equiv \frac{1}{r_j} \int_{x_j}^{\infty} (\tilde{x} - x_j) \, dG_j(\tilde{x})
\]

and note their relationship

\[
\lambda_2 + x_j \lambda_5 = 0 \quad \text{and} \quad \hat{x}_j \lambda_5 = 0.
\]

\[
(1 - \eta) q_h(\theta_h) \Gamma_h = C_h \quad \text{or} \quad (1 - \eta) \left( \frac{x^e_h - \hat{x}_h}{r_h} \right) = C_h \frac{q_h}{r_h}. \tag{1.94}
\]

which solved for \( \lambda_4 \) implies

\[
\lambda_4 = \frac{e^{-rt} C_h}{(1 - \eta) q_h(\theta_h) r_h \Gamma_h} \quad \text{and} \quad \dot{\lambda}_4 = -r \lambda_4. \tag{1.90}
\]

Inserting again in (1.89) gives

\[
\lambda_1(1 - \pi^l) - \lambda_3 = \frac{e^{-rt} C_h x_j}{(1 - \eta) q_h(\theta_h) r_h \Gamma_h} \quad \text{and}
\]

\[
\dot{\lambda}_1(1 - \pi^l) - \dot{\lambda}_3 = -r \left( \lambda_1(1 - \pi^l) - \lambda_3 \right). \tag{1.91}
\]

Proceeding analogously for \( \theta_l \) implies

\[
\lambda_2 = \frac{e^{-rt} C_l}{(1 - \eta) q_l(\theta_l) r_l \Gamma l} \quad \text{and} \quad \dot{\lambda}_2 = -r \lambda_2, \tag{1.92}
\]

\[
\lambda_5 = \frac{e^{-rt} C_l x_l}{(1 - \eta) q_l(\theta_l) r_l \Gamma l} \quad \text{and} \quad \dot{\lambda}_5 = -r \lambda_5. \tag{1.93}
\]

The optimality condition for \( y_h \) reads \( e^{-rt} - \lambda_4 (\pi^x + \pi^n) = -\dot{\lambda}_4 \). I eliminate \( \lambda_4 \) and \( \dot{\lambda}_4 \) to get the optimal job creation condition for high-skilled jobs

\[
(1 - \eta) q_h(\theta_h) \Gamma_h = C_h \quad \text{or} \quad (1 - \eta) \left( \frac{x^e_h - \hat{x}_h}{r_h} \right) = C_h \frac{q_h}{r_h}. \tag{1.94}
\]
Similarly, transforming $\frac{\partial H}{\partial y_l} = e^{-rt} - \lambda_5 (\pi^x + \pi^n + \pi^h) = -\dot{\lambda}_5$ gives the optimal low-skill job creation condition

$$(1 - \eta) q_l(\theta_l) \Gamma_l = C_l \quad \text{or} \quad (1 - \eta) \left( \frac{x_l^e - \hat{x}_l}{r_l} \right) = \frac{C_l}{q_l}.$$  \hspace{1cm} (1.95)$$

Combine those two conditions with my expressions for the co-states to get

$$\lambda_1 (1 - \pi^l) - \lambda_3 = \frac{e^{-rt} \bar{x}_h}{r_h}, \quad \lambda_2 = \frac{e^{-rt} \bar{x}_l}{r_l}, \quad \lambda_4 = \frac{e^{-rt}}{r_h}, \quad \lambda_5 = \frac{e^{-rt}}{r_l}. \hspace{1cm} (1.96)$$

Compute $\frac{\partial H}{\partial e_l} = -\dot{\lambda}_2$, eliminate all known co-states and transform to get

$$\bar{x}_l - h - \frac{\eta}{1 - \eta} C_l \theta_l + \pi^h \Gamma_h + \pi^n \Gamma_l \left( \frac{1}{e^{-rt}} \right) (1 - \pi^l) \pi^h = 0. \hspace{1cm} (1.97)$$

Note that this equation implies that $\dot{\lambda}_1 = -r \lambda_1$ and consequently $\lambda_3 = -r \lambda_3$.

Next, I calculate $\frac{\partial H}{\partial e_h} = -\dot{\lambda}_1 = r \lambda_1$ which gives

$$\lambda_1 r = -e^{-rt} \left[ C_h \theta_h - C_l \theta_l \right] + e^{-rt} \theta_h q_h(\theta_h) \Gamma_h - e^{-rt} \theta_l q_l(\theta_l) \Gamma_l. \hspace{1cm} (1.98)$$

Use the job creation conditions (1.94) and (1.95) to eliminate $q_j(\theta_j) \Gamma_j$ by $\frac{C_j}{1 - \eta}$ and rearrange to arrive at

$$\frac{\lambda_1}{e^{-rt}} = \frac{\eta}{1 - \eta} \left[ \frac{C_h \theta_h - C_l \theta_l}{\bar{r}} \right]. \hspace{1cm} (1.99)$$

Insert this expression in (1.97) to derive the optimal job destruction condition for low-skilled workers

$$\hat{x}_l - h - \frac{\eta}{1 - \eta} C_l \theta_l + \pi^h (1 - \pi^l) \frac{\eta}{1 - \eta} \left[ \frac{C_h \theta_h - C_l \theta_l}{\bar{r}} \right] + \pi^n \Gamma_l + \pi^h \Gamma_h = 0. \hspace{1cm} (1.100)$$

Compute $\frac{\partial H}{\partial e_h} = -\lambda_3$ and eliminate $-\lambda_3$ using $\lambda_3 = e^{-rt} \left(1 - \pi^l\right) \frac{\eta}{1 - \eta} \left[ \frac{C_h \theta_h - C_l \theta_l}{\bar{r}} \right]$. Rearranging reveals the optimal job destruction condition for high-skilled workers

$$\hat{x}_h - h - \frac{\eta}{1 - \eta} \left[ \frac{(1 - \pi^l) r}{\bar{r}} C_h \theta_h + \frac{\pi^l (1 + r)}{\bar{r}} C_l \theta_l \right] + \pi^n \Gamma_h = 0. \hspace{1cm} (1.101)$$

Observe how $\pi^l = \pi^h = 0$ make the conditions collapse to their intragroup forms as derived in appendix section 1.D.
1.F More comparative statics for the intragroup model

JD-JC diagram

Note that the determinant of the Jacobian of the JD-JC system is always positive, as $JD_\theta \equiv \frac{\partial JD}{\partial \theta} < 0$, $JD_\hat{x} \equiv \frac{\partial JD}{\partial \hat{x}} > 0$, $JC_\theta \equiv \frac{\partial JC}{\partial \theta} < 0$, and $JC_\hat{x} \equiv \frac{\partial JC}{\partial \hat{x}} < 0$, i.e.

$\text{Det}(JDJC) = JD_\theta JC_\hat{x} - JC_\theta JD_\hat{x} = \nabla > 0$. The elements of the inverse of the Jacobian of $JDJC$ system have the following signs

$$\left( Jac_{JDJC} \right)^{-1} = \nabla^{-1} \begin{pmatrix} JC_\hat{x} & -JD_\hat{x} \\ -JC_\theta & JD_\theta \end{pmatrix} = \begin{pmatrix} - & - \\ + & - \end{pmatrix}.$$ 

To prove that the JD-curve slopes upward and the JC-curve is downward sloping proceed as follows. Total differentiation of the JD-curve w.r.t. $\hat{x}$ and $\theta$ gives

$$(1-\tau)(1-G(\hat{x}))\pi^n + (1-\tau)r \frac{d\hat{x}}{\pi^n + r} = \frac{\omega c}{1-\omega}d\theta, \text{ hence} \quad \frac{d\theta}{d\hat{x}}|_{JD} > 0, \text{ the JD-curve is increasing.}$$

Before deriving the slope of the JC-curve, let me define $\frac{\partial x^e}{\partial \hat{x}} = \frac{g(x)}{1-G(\hat{x})} = \Psi$.

**Assumption 1.4.1.** $\Psi < 1$. This is true in any case for some distributions (e.g. uniform, normal, . . . ) and very likely to be true for others (e.g. log-normal, with sufficiently small variance)\(^{36}\).

Again, total differentiation reveals that

$$\left[ \frac{(1-\tau)(1-\omega)}{\pi^n + r}(\Psi - 1) - \frac{c}{q^f}S(\hat{x})q(\theta) \right] \frac{d\hat{x}}{\Psi} = \frac{\eta c}{q^o}d\theta,$$

$$\frac{d\theta}{d\hat{x}}|_{JC} < 0, \text{ the JC-curve is decreasing.}$$

**Policy effects**

If total effects are not mentioned, it means that they are ambiguous.

**Wage subsidy (D)**

$$\frac{d\theta}{dD}|_{JD} = \frac{1-\omega}{\omega c} > 0 \quad \text{and} \quad \frac{d\theta}{dD}|_{JC} = 0.$$

\(^{36}\)It is easy to analytically show that for the uniform distribution $\Psi = 1/2, \forall \hat{x}$. Statements about the other distributions are based on numerical simulations.
Effect: The JD-curve shifts outward. The JC-curve does not move. \( \theta \uparrow, \hat{x} = x \downarrow, u \downarrow. \)

\textbf{Hiring subsidy (H)}

\[
\frac{d\theta}{dH} \bigg|_{JD} = 0 \quad \text{and} \quad \frac{d\theta}{dH} \bigg|_{JC} = -\frac{q^w(1 - \omega)[\Psi - 1]}{\eta c} > 0.
\]

Effect: The JD-curve does not move. The JC-curve shifts outward. \( \theta \uparrow, \hat{x} \uparrow, x < \hat{x}. \) To determine the effect on the direction of \( x \) see proposition 1.2.1.

\textbf{Recruitment subsidy (R)}

\[
\frac{d\theta}{dR} \bigg|_{JD} = \frac{\theta}{c} > 0 \quad \text{and} \quad \frac{d\theta}{dR} \bigg|_{JC} = \frac{\theta}{\eta c} > 0.
\]

Effect: The JD- and the JC-curves shift outward. \( \theta \uparrow. \) As the JC-curve moves stronger this implies that \( \hat{x} = x \uparrow. \)

\textbf{Firing tax (F)}

\[
\frac{d\theta}{dF} \bigg|_{JD} = \frac{(1 - \omega)r}{\omega c} > 0 \quad \text{and} \quad \frac{d\theta}{dF} \bigg|_{JC} = \frac{q^w(1 - \omega)[\Psi - 1]}{\eta c} < 0.
\]

Effect: The JD-curve shifts outward and the JC-curve shifts inward. \( \hat{x} \downarrow, x > \hat{x}. \) Using the implicit function theorem one can show that \( \theta \downarrow. \)

\textbf{Output taxes (\( \tau \))}

\[
\frac{d\theta}{d\tau} \bigg|_{JD} = -\frac{(1 - \omega)}{\omega c} \left[ (x^e - \hat{x})(1 - G(\hat{x}))\pi^n + (\pi^n + r)\hat{x} \right] < 0.
\]

\[
\frac{d\theta}{d\tau} \bigg|_{JC} = -\frac{q^w(1 - \omega)}{(1 - \tau)(\pi^n + r)\eta c} \left[ (x^e - \hat{x})(1 - \tau) - \Psi(F - H)(\pi^n + r) \right] < 0.
\]

This expression is smaller than 0, i.e. the JC shifts inward, whenever \( F = H. \) The bigger \( F \) in comparison to \( H, \) the smaller the inward shift.

Effect: The JD- and the JC-curves shift inward. \( \theta \downarrow. \)

\textbf{Payroll taxes (t)}

\[
\frac{d\theta}{dt} \bigg|_{JD} = -\frac{z(1 - \omega)}{(1 - t)^2 \omega c} < 0 \quad \text{and} \quad \frac{d\theta}{dt} \bigg|_{JC} = 0.
\]

Effect: The JD-curve shifts inward. The JC does not move, implying \( \theta \downarrow, \hat{x} \uparrow. \)
1.G Tables

Table 1.4: Variable names

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j = l, h$</td>
<td>subscript indicating the skill type</td>
</tr>
<tr>
<td>$\hat{}$</td>
<td>hat notation refers to 'inside'-variables</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>average productivity</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>average wage</td>
</tr>
<tr>
<td>$\omega$</td>
<td>bargaining weight for the worker</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>bound (lower) for $G(\cdot)$</td>
</tr>
<tr>
<td>$\bar{\kappa}$</td>
<td>bound (upper) for $G(\cdot)$</td>
</tr>
<tr>
<td>$G(x)$</td>
<td>cdf for productivity draws</td>
</tr>
<tr>
<td>$X^c$</td>
<td>conditional expectation of some random variable $X$ w.r.t. $x$</td>
</tr>
<tr>
<td>$X^\hat{c}$</td>
<td>conditional expectation of some random variable $X$ w.r.t. $\hat{x}$</td>
</tr>
<tr>
<td>$\nabla$</td>
<td>determinant of the JD-JC system</td>
</tr>
<tr>
<td>$F$</td>
<td>firing taxes</td>
</tr>
<tr>
<td>$H$</td>
<td>hiring subsidy</td>
</tr>
<tr>
<td>$h$</td>
<td>home production</td>
</tr>
<tr>
<td>$\mu$</td>
<td>instantaneous value of non-work ($\mu = b + h$)</td>
</tr>
<tr>
<td>$r$</td>
<td>interest rate</td>
</tr>
<tr>
<td>$L$</td>
<td>labor force</td>
</tr>
<tr>
<td>$\theta$</td>
<td>labor market tightness</td>
</tr>
<tr>
<td>$e$</td>
<td>mass of employed people</td>
</tr>
<tr>
<td>$u$</td>
<td>mass of unemployed people</td>
</tr>
<tr>
<td>$\bar{G}(x)$</td>
<td>partial expectation of productivity</td>
</tr>
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<td>$g(x)$</td>
<td>pdf for productivity draws</td>
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<td>$\pi^l$</td>
<td>prob. of downgrade</td>
</tr>
<tr>
<td>$\pi^x$</td>
<td>prob. of exogenous separation</td>
</tr>
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<td>$q^f$</td>
<td>prob. of filling a vacancy</td>
</tr>
<tr>
<td>$q^w$</td>
<td>prob. of finding and accepting a job</td>
</tr>
<tr>
<td>$q(\theta)$</td>
<td>prob. of match for the firm</td>
</tr>
<tr>
<td>$\theta q(\theta)$</td>
<td>prob. of match for the worker</td>
</tr>
<tr>
<td>$\pi^n$</td>
<td>prob. of new productivity draw</td>
</tr>
<tr>
<td>$\pi^h$</td>
<td>prob. of upgrade</td>
</tr>
<tr>
<td>$x$</td>
<td>productivity</td>
</tr>
<tr>
<td>$R$</td>
<td>recruitment subsidy</td>
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<tr>
<td>$\bar{x}$</td>
<td>reservation productivity, 'outside'</td>
</tr>
<tr>
<td>$\hat{x}$</td>
<td>reservation productivity, 'inside'</td>
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<td>$\Omega$</td>
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<td>surplus function</td>
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<td>total production</td>
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<td>output tax rate</td>
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<td>$b$</td>
<td>unemployment compensation</td>
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<tr>
<td>$C$</td>
<td>vacancy creation costs (gross)</td>
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<tr>
<td>$c$</td>
<td>vacancy creation costs (net of subsidies, i.e. $c = C - R$)</td>
</tr>
<tr>
<td>$U$</td>
<td>value of a being unemployed</td>
</tr>
<tr>
<td>$J(x)$</td>
<td>value of employment for the firm</td>
</tr>
<tr>
<td>$W(x)$</td>
<td>value of employment for the worker</td>
</tr>
<tr>
<td>$V$</td>
<td>value of a vacancy</td>
</tr>
<tr>
<td>$D$</td>
<td>wage subsidy (lump-sum)</td>
</tr>
<tr>
<td>$t$</td>
<td>payroll tax rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>weight in the matching function</td>
</tr>
</tbody>
</table>
Chapter 2

Employment Protection, Labor Market Turnover and the Effects of Globalization

Philip Schuster†

†The paper was presented at seminars at the University of St. Gallen, the Spring Meeting of Young Economists (2011, Groningen), the annual conference of the Association for Public Economic Theory (2011, Bloomington, IN) and the annual meeting of the German Economic Association (2011, Frankfurt). I thank the participants for helpful comments and discussions. In particular I thank Stefan Bühler, Gabriel Felbermayr, Reto Föllmi, Christian Keuschnigg, Jochen Mankart, and Giovanni Mellace for their advice. All remaining errors are my own.
2.1 Introduction

Recent work has emphasized the need for understanding the role of the welfare state and social protection in a world of increasing economic integration. So far these papers have predominantly focused on unemployment insurance (see e.g. Keuschnigg and Ribi, 2009). This work extends the analysis by looking at the role of employment protection (EP) in an open economy. A parsimonious static version of the Diamond-Mortensen-Pissarides (DMP) model with one-shot worker-firm matching, bilateral wage bargaining and endogenous separation is enriched to capture effects of trade through changes in marginal revenues which are usually assumed to be exogenous and constant in the canonical DMP model. While most studies that integrate modern trade models (e.g. Melitz, 2003) with theories of frictional labor markets focus on unemployment levels, the developed model is especially equipped to analyze labor market turnover as an important channel connecting EP and trade. This is important as the same level of unemployment can be generated by different compositions of job creation and destruction. The specific question the paper seeks to answer is how the effects of EP are altered after arrival of a trade liberalization shock in a North-North trade setting. In addition the optimal implementation of EP is discussed.

In principle, the causal relationship of labor market institutions and trade patterns can be two-fold. Papers, like Cuñat and Melitz (2007) and Helpman and Itskhoki (2010) describe how a flexible labor market implies a comparative advantage in producing in high-volatility sectors. Closely related, Davidson et al. (1999) explain how country-specific differences in sectoral labor turnover rates determine trade. This paper is related to another stream of the literature that analyzes how trade patterns shape labor market institutions and social protection. Most attention concerning the role of social protection in an open economy was put on unemployment insurance (UI). EP, as another pillar of the welfare state, and its interaction with globalization shocks has received comparatively little attention so far. But what is a potential role of EP on top of UI? Blanchard and Tirole (2008) argue that existing UI creates a firing externality that can be undone by introducing EP in form of a firing tax. It is well understood that EP reduces both job destruction as well as job creation (see for example Messina and Vallanti (2007) for empirical support). While the effect on employment of locally increasing EP from a small level is ambiguous, it is clear that ever increasing EP will eventually lead to a reduction in employment, which char-
characterizes the downside of EP. Hence, an optimally chosen EP efficiently trades off the positive effect of correcting the firing externality and the negative effect of a reduction in the level of employment. This trade-off will be picked up in the normative part of the paper. This study also features a positive part dealing with the effects on welfare for a given level of firing restrictions.

That increased international integration should lead to more volatility in employment has been widely argued in the literature (see for example Rodrik, 1997, or Bhagwati and Dehejia, 1994) although there are hardly any formalizations of this idea. A typical argument is that increased opportunities to trade in intermediate goods make labor demand of domestic final good producers more elastic as they can more easily switch suppliers and source from abroad. I explicitly model a channel which implies that whenever opening to trade leads to an increase in ‘chances’ and ‘threats’ for domestic firms, job creation and destruction will rise. Chances can be thought of as new export markets while threats can come from import competition. I will make use of a production technology similar to the idea of trade in tasks by Grossman and Rossi-Hansberg (2008). While there is limited substitutability between domestic tasks or intermediate goods, i.e. final output cannot be increased by simply repeating same task, a domestic variety\(^1\) could be perfectly replaced by a foreign one from the same industry. Imagine assembling a car and how a German car engine cannot be substituted by adding a second German steering wheel but by a French car engine. In contrast to their framework I do not apply the technology assumption in an off-shoring- or North-South-context, where one country has a persistent cost advantage in which case, given the additional option of cheap sourcing from the South, Northern firms unambiguously profit and generate a clear positive productivity effect. As already hinted by the car example, I will focus on a perfectly symmetric North-North set-up where the production advantage is stochastic and driven by idiosyncratic shocks that are uncorrelated between sectors. Hence, in one sector a firm can steal business from the corresponding competitor in the other country, while this pattern could be reversed in another sector. As a domestic firm is ex-ante unaware of whether it can gain revenues, or if it will lose market shares, the distribution of potential revenues and hence profitability widens which will be the key determinant for a trade-induced rise in labor turnover. This kind of spread is also present in new trade theory models à la Melitz (2003) that predict a reallocation of market shares from low- to

\(^1\)The terms ‘task’, ‘intermediate good’, and ‘variety’ have the same meaning in this context and are used interchangeably.
high-productivity firms\(^2\) while the link to flows between the pools of employed and unemployed workers is absent in those frameworks. The idea that openness to trade can amplify the ‘winner-loser’-pattern within an industry is well established empirically (see for example Pavcnik, 2002, Tybout, 2003, Bernard, 2004, and Baggs, 2005).

In the parsimonious model developed in this paper job creation is directly linked to expected profits of firms. If expected profits increase, more firms will enter the labor market pushing up the probability of a worker to find a job. Job destruction is driven by match-specific shocks to productivity and revenues and therefore consequently to profitability. Hence, there is a cut-off for the value of production below which a firm will destroy a job and lay off the involved worker. If trade liberalization lowers profitability in bad matches as argued above, it is clear that the mass of revenues below the cut-off increases, which consequently implies a rise in the job destruction rate. At the same time trade liberalization is supposed to increase profitability in good matches, boosting expected profits and therefore job creation because firms just care about values of production above the cut-off, i.e. profits that are actually realized. This argument makes clear how openness to trade affects job flows and labor turnover, while the effect on the level of employment, which is at the heart of the analysis in many other studies, might be much less accentuated. The interaction of trade and EP follows directly as EP, as argued before, is a policy instrument that works exactly at the labor turnover margin.

The model delivers the following results. Ceteris paribus, EP and openness to trade have opposing effects along many dimensions. EP unambiguously decreases job creation and destruction, while the opposite is true for a trade openness shock. While both entail ambiguous effects on the employment level, the output effects are clear cut. EP will always lead to less total and average net output per worker. An openness to trade shock will imply the opposite. Both ‘shocks’ make the wage distribution unambiguously more disperse which increases income inequality. But the spreads in the wage distribution are of completely different nature. In case of EP the wage distribution increases on both tails as (a) all wages are pushed up by a constant fraction generated in

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\(^2\)Note that the mechanism in Melitz (2003) works quite differently. In contrast to my framework, less productive firms do not suffer directly from increased competition as this channel is not present in the Melitz set-up given his assumptions on preferences. Instead, less productive firms are hurt by the increased labor demand of new entrants that bid up the wage on a competitive labor market, which is absent in my framework.
the process of wage bargaining and (b) more low paying jobs are operated because of decreased job destruction. Hence, the effect on the average wage is ambiguous. In case of a trade shock, the wage distribution it widened on the right tail as a direct consequence of increased volatility in potential revenues, while the cut-off on the left side is unaffected. Consequently, the average wage increases unambiguously. In the welfare analysis I first consider a benchmark environment with risk-neutral workers such that welfare and net output coincide. The normative question of optimal EP is therefore trivialized as there are no externalities present that justify firing restrictions. Concerning the positive part where I look at the effects of given firing costs, the parsimonious model set-up allows to derive a simple solution for the second best. It is shown that, for a reasonable parameterization, the welfare loss due to firing restrictions is increasing in job turnover, i.e. openness to trade. That means that while an open economy will always enjoy higher welfare, the distance to a possible first best is also increasing, i.e. open economies suffer relatively more from EP. As an extension, I introduce risk-aversion of the workers to create a firing externality in the spirit of Blanchard and Tirole (2008). Workers demand unemployment insurance which creates a fiscal externality as firms do not take into account that an UI-system has to be financed when they decide to lay off a worker. Blanchard and Tirole (2008) find that an optimal firing tax has to be positive in that case. My results differ because I explicitly model endogenous job creation. It is still true that firms do not internalize the social costs of firing, but they also do not internalize the social benefit from hiring. Therefore whether an optimal firing tax should be positive or not is tightly linked to the effect on the tax base and therefore on the level of unemployment. A short numerical example reveals that employment is likely to decrease in the present set-up and that a firing tax should be set to 0, as in the case of risk-neutral workers.

The paper further relates to other strands of the literature. Helpman et al. (2010) and Felbermayr et al. (2011) integrate frictional labor markets of DMP style into the Melitz (2003)-framework in order to analyze the effect of trade liberaliza-

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3 The typical search externalities in the labor market are assumed to be balanced (Hosios (1990)-condition) throughout the paper.

4 This has been argued before in the literature. See e.g. Coles (2008) for a similar argumentation in a model that solely focuses on the job creation margin.

5 Empirical evidence on the effect of EP on employment levels - mostly drawn from cross-country analyses - is mixed. Some papers, including the seminal work of Lazear (1990), or Nickell (1997), Heckman and Pagés (2000), and Kahn (2007) find a significant negative effect. Other studies like OECD (1999) and Addison et al. (2000) report no significant effects. Addison and Teixeira (2003) and, more recently, Skedinger (2011) provide extensive summaries of the empirical literature.
tion on inequality and the level of unemployment. Both develop thorough dynamic models incorporating firm self-selection into exporter/non-exporter-status, multi-worker firms, etc. - features that are missing in the present paper. On the other hand they do not allow for trade effects along the job destruction margin which is at the heart of this study. In addition, the parsimonious set-up allows for a more comprehensive welfare analysis. Probably the most closely related analysis was done by Jansen and Turrini (2004) which also features endogenous job destruction. They consider two ‘globalization scenarios’. First, an economy is allowed to move from autarky to symmetric two-country trade. Given their technology assumptions integration is followed by increased demand for intermediate goods which inflates prices as supply is fixed. The effect is comparable to a positive shock to total factor productivity and shifts the whole distribution of potential revenues to the right, leading to a reduction in job destruction. In a second scenario, they assume an exogenous increase in the volatility of cost-price mark-ups, as in Mortensen and Pissarides (1994), which results in an increase in job destruction. The contribution of this paper is to bring both scenarios together by showing that using a Grossman and Rossi-Hansberg (2008)-technology implies that market integration of two symmetric countries leads to a spread in potential revenues, which is more consistent with the ‘winner-loser’ pattern present in new trade theory. The model predicts an increase in job destruction and job creation, hence in job turnover. The link of openness to trade and job turnover has received moderate attention in the empirical literature, probably because suitable data on worker flows is rather limited compared to employment level data. A positive effect of openness to trade on job turnover is empirically supported by Haltiwanger et al. (2003) and Faggio and Konings (2003) for transitions economies, Haltiwanger et al. (2004) and Ribeiro et al. (2004) for trade liberalization in South America, by Groizard et al. (2010) and Kletzer (2000) for US manufacturing and Beaulieu et al. (2004) for Canada. In contrast, Klein et al. (2002) find no evidence that the establishment of NAFTA had an influence on worker flows. Brülhart et al. (1998) use Irish data to show that exposure to trade has no effect on between industry labor reallocation but a small positive effect on within industry job turnover. However, one should be careful when linking empirical results relying on industry or firm level data to the model in this paper which rather presents a theory of production plants. To a great extent international trade is carried out by multinationals and worker flows within a multinational company might not be recorded accurately, which could underestimate job turnover compared to the predictions of the model.
The remainder of the paper is organized as follows. Section 2.2 presents a simple labor market model featuring endogenous job creation and separation where the distribution of before-wage profits is taken as given. In section 2.3 I will discuss the effects of an increase in the risk of profitability on job flows. Section 2.4 presents a simple intermediate goods trade model that delivers a microfoundation for a rise in volatility of the form that was assumed in the previous section. The welfare implications of openness to trade and EP are derived in section 2.5 before section 2.6 concludes.

2.2 Labor market model

First, I analyze the labor market assuming that production is just characterized by an exogenous distribution of possible production values. This assumption will be motivated and interpreted in section 2.4. The set-up of the labor market model is static and can be summarized by the following sequence of events:

Stage 1. A mass 1 of workers starts out as unemployed.

Stage 2. Firms enter the labor market according to a free entry condition by posting one vacancy each at cost $c$.

Stage 3. Workers are hired according to a matching technology $M$.

Stage 4. A value of production $y$, i.e. a realization of a given random variable $Y$, is revealed to each firm leading to firing of the most unprofitable workers.

Stage 5. In case of separation firms have to pay firing costs $F$. Laid-off workers receive $z = h + b$ like the workers who were never hired, where $h$ denotes home production and $b$ is unemployment compensation. Production is started with the remaining workers, who receive a bargained wage $w$.

Before solving the model by backward induction I will specify the matching technology and preferences. The labor market is characterized by a typical matching function assumed to fulfilled the following conditions. The matching function $M(u, v)$ is homogeneous of degree one and increasing in its two arguments: number of initially unemployed $u$ and number of vacancies $v$. Define labor market tightness as the vacancy-unemployment ratio, i.e. $\theta \equiv \frac{v}{u} = \frac{v}{1}$. The firm’s probability of matching can be expressed as $m^f = \frac{M}{v}$ with an elasticity
of \( m^f \) w.r.t. of \(-\eta \in (-1, 0)\) which is assumed to be constant. A worker’s probability of being matched is \( m = \frac{M}{u} = \theta m^f \) with \( \frac{dm}{du} = (1 - \eta)m^f > 0 \). Hence, \( m \) is referred to as the job finding or job creation rate\(^6\) and let \( G \) be the separation or job destruction rate. Employment\(^7\) is given as the number of workers that are matched and not subsequently laid off

\[
e = m(1 - G),
\]

which evolves as follows

\[
d e = (1 - G) \, dm - m \, dG.
\]

Clearly employment is increasing the job creation rate and decreasing in the job destruction rate. Workers’ utility functions are strictly increasing, \( u'(\cdot) > 0 \). Expected utility of a worker \( i \) is given as

\[
V_i = \left\{ \begin{array}{ll}
\qquad m(1 - G) \cdot u(w_i) + (1 - m(1 - G)) \cdot u(z) \\
\end{array} \right. 
\]

Wages will differ for workers because worker-firm pairs may have different values of production \( y \). Integrating this expression over all individuals gives total utilitarian welfare \( \Omega = \int_0^1 V_i \, di \) as firms make zero profits in equilibrium. For the moment, I will focus on risk-neutral\(^8\) workers, i.e. \( u(x) = x \).

The last decision stage of the agents is the firing or separation decision. Firms will lay off workers whenever the realized value of production\(^9\) \( y \) minus labor costs \( w(y) \) is lower than the firing costs \( F \), i.e.

\[
y - w(y) < -F \Rightarrow y = w(y) - F,
\]

where \( y \) denotes the value of production at which a firm is indifferent between firing and keeping the worker. \( y \) is the realization of an i.i.d. draw from the known distribution \( G_Y(\cdot) \). Recall that \( G_Y(\cdot) \) is an endogenous object that will

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\(^{6}\)As the initial unemployment rate is equal to 1, the job finding rate \( m \) and the number of matches \( M \) give the same number.

\(^{7}\)Using mass 1 of workers implies that \( e \) also denotes the probability of being employed for a specific worker due to the law of large numbers.

\(^{8}\)This assumption is relaxed in section 2.5.3.

\(^{9}\)Readers should think of the market value of production \( y \) as ‘before-wage profits’ that capture productivity as well as the demand structure, i.e. the price and revenue that can be generated, in reduced form. The paper will be more explicit about how \( y \) is derived in section 2.4. As the distribution of match-specific productivity will be constant all changes in the distribution of \( Y \) can be interpreted as changes in the distribution of revenues.
depend on the current trade regime. As the model is set up in a way such that equilibrium is recursive one can solve the labor market taking \( G_Y(\cdot) \) as given for the moment. \( G_Y(y) \), or in short \( G \), gives the probability that a worker-firm pair is insufficiently profitable and will therefore be referred to as firing or job destruction rate. To specify \( G \) it is necessary to know the reservation wage \( w(y) \). Wages are determined using a standard bilateral Nash bargaining game

\[
w(y) = \argmax [u(w) - u(z)]^\omega [y - w + F]^{1-\omega},
\]

where \( \omega \) denotes the worker’s bargaining power. After using the assumption of risk-neutrality the first-order condition reads

\[
\frac{\omega}{w - z} = \frac{1 - \omega}{y - w + F}.
\]

Hence, the wage schedule is given by

\[
w(y) = (1 - \omega)z + \omega [y + F].
\]

Substitute the reservation wage out of the cut-off condition (2.4) to get

\[
\underline{y} = z - F.
\]

Equation (2.8) is the first central equilibrium condition and will be referred to as the job destruction condition. The intuition is that at the cut-off the surplus of forming a worker-firm pair is 0, hence the wage is pushed down to the outside option \( z \). Consequently, there will be no inefficient firing\(^{10} \) that could be prevented by bilateral trade between the firm and the worker. The job destruction rate \( G \) is just

\[
G = G_Y (z - F).
\]

After solving for the firing decision one can analyze the preceding job creation decision which is driven by the profit a firm can anticipate to make. The expected profit conditional on being matched with a worker is denoted

\[
\pi^e = (y^e - w^e) (1 - G) - GF,
\]

where \( y^e \) and \( w^e \) denote conditional expectations of \( y \), i.e. \( y^e = E(Y|y > y) = \int_y^\infty y dG_Y(y)/(1 - G) \), and \( w(y) \), i.e. \( w(y^e) \). Substitute the wage schedule (2.7)

\(^{10}\)The terms ‘firing’ and ‘separation’ can therefore be used interchangeably.
into the expected per-worker profits $\pi^e$ and rearrange to get

$$\pi^e = (1 - \omega) \left[ y^e - y \right] (1 - G) - F.$$  \tag{2.11}

Expected profits are decreasing in the outside option, the bargaining power of the worker and the firing costs. Latter can eventually lead to negative expected profits. Firms are assumed to have an outside option of 0 and will therefore enter the labor market as long as $m^f \pi^e - c \geq 0$ where $c > 0$ are costs of entering. I assume the parameters in a range such that $\pi^e > 0$ is guaranteed to avoid the uninteresting case of zero entry. As more firms enter, the tightness of the market increases which drives down the probability of being matched with a worker. In equilibrium firms will enter up to the point where there is no more gain from doing so. The free entry condition therefore states that $m^f \pi^e = c$ or

$$(1 - \omega) \left[ y^e - z + F \right] (1 - G) - F = \frac{c}{m^f}. \tag{2.12}$$

This pins down the job creation rate which is given as

$$m \equiv m(\theta) = m \left( [m^f]^{-1} \left( \frac{c}{\pi^e} \right) \right), \tag{2.13}$$

where $[m^f]^{-1} (\cdot)$ denotes the inverse function of $m^f(\cdot)$. It is easy to see that the job creation rate is increasing in $\pi^e$ and decreasing in $c$. Equilibrium in the labor market is given by the vector $\langle \theta, y \rangle$ that solves the job destruction (2.8) and the job creation condition (2.12). These conditions resemble the equilibrium conditions of a fully dynamic version of the model (e.g. Pissarides, 2000) except that expected profits are not discounted and the job destruction condition does not incorporate possible future shocks to $y$. A first result is given by the following proposition.

**Proposition 2.2.1.** An increase in firing costs leads to a reduction in both, the job creation rate (through lower labor market tightness) as well as the job destruction rate. The effect on the employment level is ambiguous.

The proof is provided in appendix 2.A. This result is well understood and intuitive. An increase in $F$ pushes the cut-off value of production $y$ up and consequently the job destruction rate falls. At the same time firing costs reduce

---

11 An alternative representation of (2.12) often used for calculations is $\epsilon(1 - \omega) \left[ y^e - z \right] - mGF - \epsilon \omega F = c \theta$.

12 For an explicit relationship one could consider a typical Cobb-Douglas specification, e.g. $m^f = M_0 \theta^{-\eta}$ and consequently $m = \theta m^f = M_0 \theta^{1-\eta}$. Then labor market tightness is given as $	heta = M_0^{1/\eta} \left[ \pi^e/c \right]^{1/\eta}$ and the job creation rate would be $m = M_0^{1/\eta} \left[ \pi^e/c \right]^{(1-\eta)/\eta}$. 

expected profits as stated in (2.11). Clearly, firms will create less vacancies and the workers’ job finding rates fall. The effect on wages is summarized in the next proposition.

Proposition 2.2.2. An increase in firing costs leads to an increase in the spread of the wage distribution. Whether the average wage falls or rises is solely determined by the nature of the underlying distribution of $Y$.

The proof is provided in appendix 2.A. The first part is intuitive as firing costs prevent firms from destructing jobs with low profitability which pay low wages. Consequently, the wage distribution is widened on the left tail. The result that this does not automatically imply a decrease in the average wage stems from the fact that all wage rise by the constant term $\omega$ that workers can snatch in the bargain for every unit of $F$.

2.3 Increase in the risk of profitability

This section discusses how an increase in the riskiness of the value of production changes the equilibrium allocation. This increase could in principle have very different motivations, like what happens in an environment where firms are forced to engage in more risky projects. Another story, told by this paper, explains how international market integration can imply increased chances as well as increased threats to local firms, hence spreading out the distribution of potential revenues of firms after integration. I will be more explicit about this in section 2.4 where a simple two-country model is used to show that international market integration implies a spread in the distribution of revenues and therefore profitability of the form that will be analyzed in a general and abstract manner in what follows now.

For simplicity and tractability I assume that the increase in volatility takes place in the simple form of a mean preserving single crossing spread from $Y$ to $Y'$.

Definition 1. A mean preserving single crossing spread (MPSCS) from $Y$ to $Y'$ is given if the following two conditions hold

a) Mean preservation (MP):

$$E(Y) = E(Y') = \int_{-\infty}^{\infty} y \, dG_Y(y) = \int_{-\infty}^{\infty} y \, dG_{Y'}(y) = \mu_Y,$$
b) Single crossing spread (SCS):

$$\exists \hat{y} : G_Y(y) \geq G_{Y'}(y), \forall y \geq \hat{y} \text{ and } G_Y(y) \leq G_{Y'}(y), \forall y \leq \hat{y}. $$

The notion of MP is self-explanatory while the SCS characteristic just implies that the cumulative distribution function (cdf) of the spread random variable $Y'$ crosses the cdf of $Y$ just once from above at the intersection point\(^{13}\) $\hat{y}$ as illustrated in figure 2.1 in appendix 2.F. Assuming this, one can analyze how job flows are affected but such a volatility increase.

**Lemma 2.3.1.** The job destruction rate $G$ is weakly increasing for any kind of SCS of $Y$ if $y \leq \hat{y}$.

**Proof.** This follows directly from the SCS definition. ■

The intuition for this result is clear and also illustrated in figure 2.1. While the cut-off $\underline{y}$ is unaffected, a bigger mass is now located below it, leading to a higher probability of separation. Let me address the effect on job creation now.

**Lemma 2.3.2.** The job creation rate $m$ is weakly increasing for any kind of MPSCS of $Y$.

The proof is provided in appendix 2.A. The basic intuition is again simple to grasp and best understood by looking at figure 2.2. While the unconditional expectations of both distributions are the same, only the revenues above the cut-off $\underline{y}$ are actually realized. This means that the increase in the mass of low profitability does not hurt the firm because those jobs would not have been operated anyways while the firm profits from a higher probability of drawing a high revenue. This pushes up expected profits and consequently the job creation rate $m$. Hence, labor turnover is accelerated. Note the analogy to the effect of EP just with opposite direction. As in the case of EP the effect of a revenue spread on employment is undetermined while effects on welfare are clear cut as will be shown in section 2.5. The following corollary gives some supplement results.

**Corollary 2.3.1.** Average output $y^e$, average wage $w^e$ and the wage dispersion are weakly increasing for any kind of MPSCS of $Y$ if $y \leq \hat{y}$.

The proof is provided in appendix 2.A. If one wants to compute the exact changes in $G$ and $m$, e.g. to be able to sign the change in employment, one has to be more specific about the nature of the MPSCS. The next section develops

\(^{13}\) Note that the single intersection point $\hat{y}$ coincides with the mean $\mu_Y$ if the distribution is symmetric.
a microfoundation that characterizes the relation between openness to intermediate goods trade and the spread of potential values of production which is explicitly derived.

2.4 Market integration and revenue risk

The effect of an increase in the risk of revenues and therefore profitability was discussed in the last section. This part of the paper presents a simple stylized extension to the labor market model described in section 2.2 that explicitly models a channel through which openness to trade affects the distribution of firms’ revenues. So far the random variable $Y$, summarizing the distribution of potential values of production, was taken as given. More model structure will be put on the production side of the economy now. First, I will discuss this for the closed economy before allowing for free trade in intermediates with a symmetric second country. Motivation for increasing product market integration is obviously manifold. One could think of it as a result of increased standardization of intermediate goods or a removal of or reduction in prohibitive non-tariff trade barriers and so forth.

2.4.1 Closed economy

While the value of production was simply denoted $y$ and followed a given distribution so far, I am more explicit about this now. Production in the economy occurs in a continuum of intermediate good sectors and a final good sector. Every worker-firm pair from before produces a different intermediate good or variety. Hence, due to the identity: 1 worker, 1 firm, 1 variety, one can index them all with $i \in [0,1]$. As every firm represents a whole domestic industry and revenues across domestic industries will not be correlated, the assumption in the reduced model from before that every firm receives an i.i.d. draw of $y$ will not be violated. Further, one can discuss the demand structure in one industry in isolation and I will therefore drop the index $i$ if appropriate. In a nutshell the decision structure looks as follows. The representative final good producer takes prices as given and chooses the amount of input of every variety, $q_i^d$. An intermediate good producer $i$ has monopoly power in market $i$ and sets price $p_i$ and picks the optimal quantity $q_i^s$ from the demand correspondence $q_i^d$. In equilibrium demand has to equal supply in every intermediate goods market, hence $\exists q_i : q_i \in q_i^d \cap q_i^s, \forall i$. Further, demand has to equal supply in the final good market, i.e. $Q^d = Q^s$. The final good is an all-purpose good that is used
for consumption of the workers and for covering the firms’ costs and will serve
as numéraire, hence \( P = 1 \). Technologies and optimal decisions are explained
in more detail now.

The final good sector is characterized by a representative competitive firm that
uses no labor but only the varieties as inputs to assemble them using a simple
1:1 technology. The production function is non-homothetic and given by

\[
Q^g = \int_0^1 \min \left\{ 1, q^d_i \right\} \, di. \tag{2.14}
\]

The production technology captures ‘love of variety’. In principle varieties are
perfect substitutes but as technology is characterized by decreasing marginal
productivity the final good producer will never want to source all inputs from
just one variety producer. This technology assumption deviates from the typi-
cal homothetic, constant elasticity of substitution production/utility functions
used by Melitz (2003) and others. The technology choice relates to Grossman
and Rossi-Hansberg (2008)’s idea of production in tasks, where every task
has to be done exactly once\(^{14}\) to produce output. It is also closely related to
frameworks with non-homothetic preferences where consumers simply decide
whether to buy or not to buy a differentiated good. This type of 0-1-preferences
are for example discussed in Murphy et al. (1989), Matsuyama (2000), and
Foellmi and Zweimüller (2006). The optimization problem of the final good
producer is given as

\[
\max_{q^d_i \in [0,1]} \Pi = \max_{q^d_i \in [0,1]} P \cdot \int_0^1 q^d_i \, di - \int_0^1 p_i \cdot q^d_i \, di. \tag{2.15}
\]

As long as \( p_i \leq P \) the final good producer will demand any amount up to 1.
Hence, the demand or willingness to pay for every variety is a step function
horizontal at \( P \) up to \( q_i = 1 \) and then dropping to zero. As the intermedi-
ate good producers have monopoly power in every intermediates good market
they will set \( p_i = P \) and seize all the rents leaving the final good producer
with zero profits. The optimal output is the maximum output that still finds
demand, i.e. \( q^g_i = 1 \), which gives revenues of \( r_i = p_i q_i = 1 \).

I will now illustrate the decision problem of the intermediate good producer

\(^{14}\)Note that in my framework not every single ‘task’ has to be done, or in this framework’s inter-
pretation: not every single ‘variety’ has to be produced, to get positive output of the final good. This
assumption is obviously necessary as some of the intermediate firms will not produce at all which
automatically implies that \( q^g_i = 0 \).
in more detail. As the problem is the same for all intermediate good firms I will drop the firm index. An intermediate firm produces output by using the production factors labor and capital. Labor input is discrete. Either a worker is employed or not. The hiring decision is made before any revenue or cost/productivity shock materializes. Once hired a firm cannot adjust its labor input, except for the possibility of completely laying off its worker again, and is therefore more or less profitable. Capital goods, which are expressed in terms of the final good, can be thought of as machines that can be flexibly scaled up and down at constant marginal costs \( k \). The more machines are used the more output is produced although in any case only a single worker is required to supervise or handle the machines. Hence, the marginal productivity of a worker is increasing and total factor costs are decreasing in output size. The production function is given as

\[
q^s = \begin{cases} 
0 & \text{if no worker is employed,} \\
x \cdot n & \text{if a worker is employed,}
\end{cases}
\]

where \( n \) denotes the quality of production which is an i.i.d. draw from \( G_N(\cdot) \) with support bounded from below by \( k \). \( x \) are the units of capital goods that are needed to produce \( q^s \) units of the intermediate good with quality \( n \). Hence, for a low quality the intermediate good firm has to use more machines to get the same quality-adjusted output. The revenue is \( r \equiv p \cdot x \cdot n \) and before-wage profits are \( y = r - x \cdot k \). Hence, there are clear opportunities costs of scaling up production as marginal costs of producing an additional unit of \( q \) are \( k/n \). Observe that the quality shock is inversely related to the marginal costs and could be alternatively modeled as a stochastic cost shock. Before-wage profits \( y \) are therefore

\[
y = p \cdot x \cdot n - x \cdot k = q^s \left( p - \frac{k}{n} \right) = q^s \cdot \phi,
\]

where the mark-up on capital costs is denoted \( \phi \equiv p - \frac{k}{n} \). Recall that condi-

---

15See Caballero (2007) for a discussion of specificity of inputs.
16Importantly, although I assume that a single workers can operate more than one machine she can do so only within her firm or industry. She cannot handle different machines producing two or more varieties at the same time.
17This is just a simplifying assumption which guarantees that in equilibrium all mark-ups will be non-negative. This simplifies the analytic treatment but is of no qualitative importance.
18Another interpretation would be that the quality parameter gives the amount of malfunctioning or sub-standard goods that will not be accepted by the final good producer.
19In principle, the match-specific shock could be arbitrarily interpreted as quality, cost, mark-up, or productivity shock without any consequences for the analytical treatment.
tional profits including wages are \( y - w(y) \). Given the assumption that wages are bargained it is easy to see that maximizing \( y - w(y) \) is equivalent to maximizing \( y \). Hence, the optimization problem of the intermediate firm is

\[
\max_{p,q^s} q^s \left( p - \frac{k}{n} \right) \tag{2.18}
\]

subject to \( q^s \in q^d \) where

\[
q^d = \begin{cases} 
[0,1] & \text{if } p \leq P, \\
0 & \text{otherwise}. 
\end{cases} \tag{2.19}
\]

As explained before the optimal choice\(^{20}\) is \( p = q^s = 1 \). Therefore the mark-up is given by \( \phi = 1 - \frac{k}{n} \) and distributed according to \( G_\phi \) which is a simple transformation\(^{21}\) of \( G_N \). As \( q \) is always 1, before-wage profits are simply given by the random variable \( Y = \Phi \), with \( E(Y) = E(\Phi) \equiv \mu_\Phi \) and \( Var(Y) = Var(\Phi) \equiv \sigma^2_\Phi \), such that density and distribution are just

\[
g_Y(y) = g_\Phi(y) \quad \text{and} \quad G_Y(y) = G_\Phi(y). \tag{2.20}
\]

As before one can solve the labor market model just by inserting (2.20) in Stage 4.

### 2.4.2 Open economy

I will now allow international exposure to have an effect on the labor market. The effect is propagated through the product markets, namely through the potential revenues firms can make. Assume that there are a 'home' and 'foreign' country indexed by \( H \) and \( F \). Both are symmetric in every aspect\(^{22}\). In contrast to typical new trade models the integration of both countries does not imply

\(^{20}\)Recall that maximized profits are only realized if in addition the non-negative profit condition is fulfilled, i.e. \( y \geq y \). Otherwise the worker is laid off at firing costs \( F \).

\(^{21}\)For simplicity I directly assume a distribution for \( \Phi \) with finite support on \([0,\bar{\phi}]\) with \( \bar{\phi} \leq 1 \). Clearly, one could first define a distribution for \( N \) with support \([k,k/(1-\bar{\phi})]\) and then link the distributions according to

\[
g_\Phi(\phi) = g_N \left( \frac{k}{1-\phi} \right) \frac{k}{(1-\phi)^2} \quad \text{and} \quad G_\Phi(\phi) = G_N \left( \frac{k}{1-\phi} \right).
\]

See appendix 2.E for mathematical details.

\(^{22}\)A strong but very convenient assumption is that job matching is perfectly correlated in both countries. Hence, one does not have to consider the case that a variety producer is matched with a worker while the corresponding firm in the other country is not matched. One can therefore ignore that firms have to form expectations about how likely it is that this event will occur. Consequently, one can directly compare all potential revenues conditional on both firms in this industry being matched with a worker. Relaxing this assumption would not change the results qualitatively and simply constitute a mixture of the closed and open economy model presented here, as some firms face additional competition and some do not. As firms are ex-ante unaware of which of those two cases will occur they would still face a higher revenue risk.
that the number of potential varieties doubles. As before the technology in the final good sector is such that a home variety \( i \) cannot be substituted by a second unit of home variety \( j \), but I assumed that it is a perfect substitute for foreign variety \( i \). This is very similar to the trade in tasks framework of Grossman and Rossi-Hansberg (2008) with the difference that instead of North-South trade where the South has a systematic price advantage, this paper tells a North-North trade story where the advantage is stochastic. In one sector the domestic variety producer is the ‘strong’ firm and able to receive a higher market share by stealing business from the foreign producer, called the ‘weak’ firm, while the pattern might be reversed in another sector. I focus on the stylized limiting case of perfect international product market integration with zero transportation or market entry costs such that both final good producers can freely choose whether to source variety \( i \) from the home or the foreign country. Hence, a variety producer might end up supplying both countries or none at all. Which case prevails\(^{23}\) will depend on the outcome of a contest both firms enter. On one hand, the idea that two firms split the market in form of contest is an approximation to a more complex mechanism like Bertrand competition which would imply the same market share pattern but also would entail price effects\(^{24}\) which would considerably complicate the welfare analysis and comparisons with the closed economy solution.\(^{25}\) On the other hand, contests have been used to model competition for market shares in the literature\(^{26}\) before, see e.g. Friedman (1958), Bell et al. (1975), or Schmalensee (1976). In this context, contests are often interpreted as games of persuasive marketing or lobbying, etc. I follow this motivation and assume that firms compete in a contest in order to try and differentiate otherwise completely identical products. I will interpret the contest success function in a probabilistic way, i.e. the firm that is more successful in convincing the buyers of the ostensible superiority of its output will be able to steal market shares from the other firm. I will now explain this mechanism in more detail. The sequence of events in the redefined

\(^{23}\)The result of this section generalizes to any other mechanism that leads to a situation where product market integration implies said ‘strong firm’-‘weak firm’ patterns that are uncorrelated between industries.

\(^{24}\)With Bertrand competition firms cannot set monopoly prices \( p_i = P \) anymore. As mark-ups differ, the firm with the better shock will set a price that reduces the mark-up of the opponent to zero and will produce for both countries. As final good production is competitive this would also have an effect on the final good’s price.

\(^{25}\)Note that all welfare effects will therefore stem from a more efficient allocation of resources by exploiting increasing marginal products of single workers and opportunities of specialization. Bertrand competition would add an additional positive selection effect as only the firm with the lower marginal costs within an industry will survive.

\(^{26}\)Konrad (2009) provides a recent comprehensive survey of the contest and tournament literature.
labor market game looks as follows.

*Stages 1–3.* As before.

*Stage 4a.* A contest decides which firm will receive a higher or lower share of the market.

*Stage 4b.* Idiosyncratic mark-ups $\phi_H$ and $\phi_F$ (two i.i.d. draws from $G_\Phi(\cdot)$) are revealed to the home and the foreign producer of a variety. This implies values of production $y_H$ and $y_F$. The most unprofitable matches are destroyed and the workers are laid off.

*Stage 5.* As before.

The extensions are now described in more detail. The final good production technology is the same as before,

$$Q^s_H = Q^s_F = \int_0^1 \min \left\{ 1, q^d_{H,i} + q^d_{F,i} \right\} \, di.$$  

(2.21)

Hence, in the open economy world demand\(^{27}\), i.e. $Q^s = Q^s_H + Q^s_F$, for every variety is now $[0, 2]$. Two firms in every sector fight for the whole market and do so in form of a contest where the winning probability $\xi$ is given by the following simple, standard Tullock (1980) contest success function

$$\xi = \begin{cases} \frac{\kappa_H}{\kappa_H + \kappa_F} & \text{if } \max \{\kappa_H, \kappa_F\} > 0, \\ 1/2 & \text{otherwise,} \end{cases}$$  

(2.22)

where $\kappa_H$ denotes effort of the home firm and $\kappa_F$ of the foreign firm. Both players can exert only two levels of effort, namely $\kappa_\ell \in \{0, \bar{\kappa}\}$, $\ell = H, F$. Unused effort cannot be spent on another activity.\(^ {28}\) As the mark-up shocks have not materialized yet the prize of the contest is the profit of winning given the expected mark-up which is the same for both players. There is a unique Nash-equilibrium in pure strategies where both players exert full effort. Hence the probability of winning is $\xi = 1/2$ in which case one firm gets the whole market and the other gets nothing.\(^ {29}\) As the winning firm is now again monopolist

\(^{27}\)As both final good producers face exactly the same problem one can alternatively think of one big assembling firm with the production function $Q^s = \int_0^1 \min \left\{ 2, q^d_{H,i} + q^d_{F,i} \right\} \, di$ generating world demand for intermediate goods.

\(^{28}\)In principle it would make no difference in the open economy setting whether firms are endowed with effort or whether effort is costly in terms of the final good. The former option was chosen to isolate welfare effects that purely stem from reallocation of resources and specialization when moving from the closed to the open economy case.

\(^{29}\)Appendix 2.D discusses a generalization where firms can only steal parts of the other firm’s market.
and serves the whole market it would again set \( p_i = P \) but simply produce \( q_i^* = 2 \) leaving it with revenue of 2 while the other firm receives nothing. Once, the result of the contest is known and taking the optimal response of the intermediate producer into account, \( q \) is the realization of the following distribution conditional on employing a worker, \( Q \),

\[
q = \begin{cases} 
2 & \text{with probability } \frac{1}{2}, \\
0 & \text{with probability } \frac{1}{2}.
\end{cases}
\]  

(2.23)

Before-wage profits are again computed according to (2.17). In the open economy they are therefore given by the random variable \( Y' \) which is the product of the two independent random variables \( Q \) and \( \Phi \), i.e. \( Y' = Q \cdot \Phi \). While in the closed economy before-wage earnings\(^{30}\) \( y \) are always equal to \( \phi \), they will now be higher or lower with probability \( 1/2 \) each. A domestic firm has a potential of stealing business from a the foreign firm and raise its market share, while also the opposite could happen. As firms are ex-ante unaware of whether they will be the ‘strong’ or the ‘weak’ firm this implies an increase in revenue risk for a variety producer.

**Lemma 2.4.1.** The integration of both product markets implies that the mean of \( Y' \) is preserved while \( \text{Var}(Y') > \text{Var}(Y) \). It further implies a MPSCS from \( Y \) to \( Y' \) if the probability density function (pdf) of \( \Phi \), \( g_\Phi(\cdot) \), is non-decreasing.

The proof is provided in appendix 2.A. Again, the random variable \( Y' \) is just a simple transformation of \( \Phi \), with \( E(Y') = E(\Phi) = \text{Var}(Y) = \mu_\Phi \), \( \text{Var}(Y') = 2 \text{Var}(\Phi) + E(\Phi)^2 = 2 \text{Var}(Y) + \mu_\Phi^2 \) and the following distribution and density functions

\[
g_{Y'}(y) = \frac{g_\Phi(y/2)}{4} \quad \text{and} \quad G_{Y'}(y) = \frac{1 + G_\Phi(y/2)}{2}.
\]  

(2.24)

The assumption that \( g_\Phi(\cdot) \) is non-decreasing is a sufficient, but by no means necessary, condition for the intersection of \( G_Y(\cdot) \) and \( G_{Y'}(\cdot) \) to be unique\(^{31}\). Lemma 2.4.1 hints at the main result that international integration, as described above, indeed increases the spread of before-wage profits \( Y \) with all the discussed consequences in a way that was taken as given in the labor market analysis so far. The following proposition summarizes the main implications.

\(^{30}\)In the closed economy \( Q \) is degenerate and has a single mass point at 1.

\(^{31}\)Appendix 2.A shows that the considerably weaker condition \( g_\Phi(y/2) < 4g_\Phi(y) \) is also sufficient for single crossing.
Proposition 2.4.1. Assume that $g_{\Phi}(\cdot)$ is non-decreasing. Then the integration of both product markets leads to higher labor turnover and has ambiguous employment effects. Average and total net output, the average wage and the wage dispersion increase.

Proof. This follows directly from combining lemma 2.4.1 with lemmata 2.3.1 and 2.3.2 and corollary 2.3.1.

Increased job creation and destruction as well as the ambiguous effect on employment are direct results of the fact that market integration leads to a spread in revenues and before-wage profits as discussed in the previous section. While gross output, given my assumptions, only varies with employment, average costs decrease due to specialization. Consequently, average net output increases. The next section will reveal that also total net output has to rise unambiguously. As wages contain an element proportional to firms’ before-wage profits it is clear that the average wage goes up while the wage distribution becomes more dispersed.

2.5 Welfare analysis

This section presents the welfare analysis of the described model. First, I continue to assume that workers are risk-neutral. This gives a trivial first best implementation with no government intervention in the case of ‘balanced’ search externalities. In a second step it is analyzed how EP implies welfare losses and how these losses are amplified in open economies with increased necessity for labor reallocation while opening the economy in principle boosts welfare in absolute terms. In the last part of this section I check the robustness of these results by introducing risk-aversion and UI. The firing externality created by UI gives a motive for using EP as argued by Blanchard and Tirole (2008). It is shown that if endogenous job creation is taken into account the results derived in a risk-neutral framework are not very likely to change qualitatively.

2.5.1 First best allocation

Recall that all workers are ex-ante identical w.r.t. abilities and are assumed to always have the same value of home production. They are indexed by $i \in [0, 1]$. The social planner is subject to the search frictions and the idiosyncratic shocks in the values of production. He has to choose a sequence of wages $w_i$, unemployment benefits $b_i$, labor market tightness $\theta$ and the cut-off $y$ in order
to maximize utilitarian welfare subject to a resource constraint. Given risk-neutrality the social planner’s problem reads

$$\max_{\{w_i\},\{b_i\},\theta,y} \int_0^1 [ew_i + (1 - e)z_i] \, di,$$  \hspace{1cm} (2.25)

subject to equilibrium employment \((2.1)\) and the following resource constraint

$$\int_0^1 [ew_i + (1 - e)b_i] \, di = eye - c\theta. \hspace{1cm} (2.26)$$

This implies

$$\begin{align*}
(1 - \eta) \left[ y^e - h \right] (1 - G) &= \frac{c}{m^f}, \hspace{1cm} (2.27) \\
\underline{y} &= h. \hspace{1cm} (2.28)
\end{align*}$$

Let us compare these conditions with the decentralized equilibrium conditions

$$\begin{align*}
(1 - \omega) \left[ y^e - z + F \right] (1 - G) - F &= \frac{c}{m^f}, \hspace{1cm} (2.29) \\
\underline{y} &= z - F. \hspace{1cm} (2.30)
\end{align*}$$

First note that because of risk-neutrality the optimal UI is zero, i.e. \(b = 0\). Second, it is clear that one requires \(F = 0\) for implementation of the first best, given that the Hosios (1990)-condition \(\eta = \omega\) holds which I will assume. Hence, one can work with the welfare generated in a laissez-faire economy as the first best benchmark.

### 2.5.2 Second best allocation

Denote the welfare of a laissez-faire economy as \(\Omega(0)\) while the welfare in an economy with positive firing costs, \(F > 0\) is denoted \(\Omega(F)\). Defined in the equivalent (net) output terms representation\(^{32}\) this is

$$\Omega(F) \equiv ey^e + (1 - m)z + mG [z - F] - c\theta, \hspace{1cm} (2.31)$$

which is maximized subject to the equilibrium values of \(\theta,y,m,G,e\).

**Proposition 2.5.1.** Welfare is decreasing in firing costs for any non-negative level of firing costs, i.e. \(\frac{d\Omega(F)}{dF} < 0\) for all \(F \geq 0\).

The proof is provided in appendix 2.A. This comes at no surprise as there is no inefficiency present that could justify \(F > 0\) as already argued in the derivation

\(^{32}\)Both, utility and output maximization coincide in the case of risk-neutrality as shown in appendix 2.B.
of the first best allocation. Proposition 2.5.1 extends this result by showing that welfare is monotonically decreasing for all non-negative values of $F$. I will analyze the effect of $F$ on welfare in an environment that is characterized by additional inefficiencies in section 2.5.3.

Proposition 2.5.2. Welfare is weakly increasing for any MPSCS of $Y$.

The proof is provided in appendix 2.A. Note that proposition 2.5.2 even holds if a MPSCS leads to a fall in employment which is always overcompensated by the increase in average output.

Proposition 2.5.3. The welfare loss due to firing costs is weakly increasing in job turnover if $\eta = \omega \leq 1/2$.

The proof is provided in appendix 2.A. The key message is that the welfare loss of EP is particularly severe for economies with high firing costs in combination with a high matching elasticity i.e. if the number of matches is more responsive to an increase in vacancies (small $\eta$). Note that proposition 2.5.3 states a weak condition for the welfare loss to increase in job turnover. The same can be true for considerably higher levels of $\eta$ and $\omega$. Given the results of the previous section this implies that an integrated or open economy suffers relatively more in efficiency terms from firing restrictions. It still enjoys higher welfare than a closed or less open economy in absolute terms, while the distance to a possible first best rises. Firing restrictions can hence lead to a substantial failure in reaping the possible welfare gains that could result from economic integration by preventing necessary labor reallocation. Or put differently, while closed economies will suffer from firing costs, the problem becomes even more severe for open economies.

2.5.3 Risk-aversion, unemployment insurance, and firing externalities

In this section I relax the assumption of risk-neutrality and impose risk-aversion. This implies that UI should be positive which creates a firing externality as described in Blanchard and Tirole (2008). Firms do not internalize the costs created by an UI system when they decide to lay off a worker. This externality can be counteracted by EP in form of a firing tax. Before the welfare analysis is presented it is discussed how equilibrium is affected by the introduction of risk-aversion, implying $u'(x) > 0$ and $u''(x) < 0$. Note that the wage bargaining
condition changes as follows
\[ \frac{\omega}{u(w) - u(z)} u'(w) = \frac{1 - \omega}{y - w + F}. \] (2.32)

One can first-order Taylor approximate \( u(x) \) around \( w \) and evaluate the function at \( x = z \) which gives
\[ u(w) - u(z) \approx u'(w)(w - z). \] (2.33)

Using this handy approximation results in the same wage schedule as before
\[ w(y) = (1 - \omega)z + \omega [y + F]. \] (2.34)

Hence, the equilibrium allocation is again determined by
\[ (1 - \omega) [y^e - z + F] (1 - G) - F = \frac{c}{m^T}, \] (2.35)
\[ y = z - F. \] (2.36)

This conveniently implies that the employment and output level is independent of the degree of risk-aversion. Consequently, propositions 2.5.1 to 2.5.3 are also true in the environment with risk-aversion if the terms ‘welfare’, defined as the sum of workers’ utilities, is replaced by ‘output’. This distinction is important as the tasks of output and welfare maximization do not coincide anymore. Hence, in contrast to output, welfare will depend on the degree of risk-aversion. The subsequent part of this section discusses the optimization of welfare.

**First best allocation**

In case of risk-aversion the social planner’s problem reads
\[ \max_{\{w_i\},\{b_i\},\theta,y} \int_0^1 [eu(w_i) + (1 - e)u(z_i)] \, di, \] (2.37)
subject to equilibrium employment (2.1) and the resource constraint (2.26). The first-order conditions for \( w_i \) and \( b_i \) state that
\[ u'(w_i) = \lambda = u'(z_i), \] (2.38)
where \( \lambda \) is the Lagrange multiplier. (2.38) has two implications. First every employed worker receives the same wage, i.e. \( w_i = w \), and every unemployed receives the same benefits \( b_i = b \). Second, there is full insurance, i.e. \( w = z = \)
b + h. Inserting the full insurance result in the first-order conditions for $\theta$ and $y$ results in

$$(1 - \eta) [y^e - h] (1 - G) = \frac{c}{m^f},$$

$$y = h.$$  \hspace{1cm} (2.39) \hspace{1cm} (2.40)

Let us consider a possible first best implementation now. Note by comparing (2.36) and (2.40) that optimal job destruction requires $b = F$ (as in Blanchard and Tirole (2008)). But observe how $b = F > 0$ will always lead to inefficiently low job creation which is not present in Blanchard and Tirole (2008). The only first best allocation would be given by $b = F = 0$ and $w = h$. This raises one problem. $w = h$ is incompatible with Nash bargained wages\footnote{See Michau (2011) for a detailed discussion of this issue in a dynamic setting.}. Hence, there does not exist a decentralized implementation of the first best allocation.

**Second best allocation**

Let us look for a second best allocation, i.e. the market solution that maximizes welfare, now. To allow for more flexibility an additional financing instrument in form of simple lump-sum taxes\footnote{This implies that the gross wage $w(y)$ is independent of $T$ and is still given by 2.7.} $T$ is introduced. Note that in the Blanchard and Tirole (2008) framework without endogenous job creation this would not change the result that a firing tax should be used to internalize the firing externality which otherwise leads firms to destruct jobs excessively. The problem changes as follows

$$\Omega(F) \equiv \max_b m \int_y^\infty u(w(y) - T) dG_Y(y) + (1 - e)u(z - T),$$

subject to the equilibrium values of $\theta, y, w(y), m, G,$ and $e$ and the budget constraint where I assume that $F$ can be collected as a firing tax at a share $\psi \in [0, 1]$

$$T = b(1 - e) - mGF\psi.$$ \hspace{1cm} (2.42)

I will investigate the effect of introducing EP, i.e. by checking how $\frac{d\Omega(F)}{dF}$ is signed at $F = 0$ which simplifies (2.62). There are two cases: (a) if $d\Omega/dF > 0$ at $F = 0$ then $F^* > 0$ or if $d\Omega/dF < 0$ at $F = 0$ then $F^* = 0$. The results are summarized by the following proposition.

**Proposition 2.5.4.** Let the following condition be fulfilled

$$\Gamma \equiv \omega u'(\tilde{w})^e + (1 - e)u'(\tilde{z}) mG\psi - mg(y) [u(\tilde{w})^e - u(\tilde{z})] > 0.$$
2.6. CONCLUSION

Then \( \frac{de}{dF} > (\) 0 is a sufficient (necessary) condition for \( F^* > (=) 0 \). Otherwise it is a necessary (sufficient) condition for \( F^* > (=) 0 \).

The proof is provided in appendix 2.A. Observe how tightly related the effect on welfare is to the effect on employment \( \frac{de}{dF} \). The proposition states that the effect of marginally raising \( F \) on total welfare is in principle ambiguous, even at \( F = 0 \). Why is this result different from Blanchard and Tirole (2008) where one would have \( \frac{d\Omega}{dF} \big|_{F=0} > 0 \)? The main difference is the endogenous job creation margin. It is still true that firms do not internalize the social costs of firing, but they also do not internalize the social benefit from hiring. The intuition is that it is the total amount of benefits that matters, which is proportional to \( 1 - e \). The externality is created by the UI system that has to be financed which is not taken into account by the firms. Intuitively the externality is getting more severe if \( F \) leads to an increase in unemployment, which explains the tight link of the effect on welfare and the effect on employment. Whether firing taxes should be used at all hence depends on whether the condition in proposition 2.5.4 is fulfilled and especially on the effect on employment.

I will address this ambiguity with a small numerical example\(^{35} \) to get a hint which of both cases seems to be more plausible. The functional forms and parameters were chosen in accordance with the literature and to replicate an unemployment rate of \( 1 - e = 0.1 \). The choice of UI \( b = 0.1 \) was determined by welfare maximization according to (2.41) and represents a gross replacement rate of \( \frac{b}{w_e} = 0.3 \). The parameters and results are shown in table 2.1. Starting from \( F = 0 \) the model suggests that employment is decreasing in \( F \) for the chosen calibration.\(^{36} \) Although \( \Gamma > 0 \) the numerical example suggests that welfare is decreasing in \( F \) and consequently optimal EP is equal to zero. Hence, the result that EP leads to a welfare loss is likely to hold also in the case of risk-aversion as before when workers were risk-neutral.

### 2.6 Conclusion

A parsimonious static version of the Diamond-Mortensen-Pissarides (DMP) model with endogenous job creation and job destruction is combined with a

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\(^{35}\)The results of this calibrated static model should be interpreted with great care, as the model was mainly designed to derive qualitative results. For a more realistic assessment concerning the magnitude of the featured effects one should employ a full dynamic version of the model, which is left for future research.

\(^{36}\)This is in line with the literature as discussed in section 2.1, as the studies that find a significant effect almost exclusively report it to be negative.
North-North intermediate goods trade framework. The effects of openness to trade are propagated to the labor market through changes in marginal revenues which are normally assumed to be constant in the canonical DMP model. The final good production technology is such that an intermediate from a different industry has to be used to add additional value to the output, while a foreign variety is a perfect substitute for a home variety within the same industry. International product market integration implies that within an industry, consisting now of a home and foreign intermediate good producer, market shares and revenues are redistributed from the less to the more successful firm. As a firm is ex-ante unaware of its relative advantage over its competitor, openness to intermediate goods trade increases the spread of the distribution of revenues and profits it can expect. The intuition is that a domestic firm has to form expectations not only about its own productivity as in the closed economy benchmark but also about the relative advantage over the competing rival firm abroad and the consequences for market shares. The increased risk in profitability leads to more job creation, more job destruction, higher output, welfare, and wage inequality while the effect on employment is ambiguous. It is shown that the effects of international market integration are qualitatively identical to a reduction in employment protection in form of costly firing restrictions, except for wage inequality which would decrease. Further, the positive welfare effects of opening to trade are decreasing in the level of firing costs which render firing restrictions more severe for open economies by preventing necessary labor re-allocation.

Some further concluding comments are in order. First, the trade shock was analyzed by comparing the limiting cases of a closed versus a trade-friction-free open economy. Hence, the presented model is ignorant about in-between cases of gradual trade liberalization. Although not formalized, an according extension could look as follows. Suppose that trade costs entered the model. The price in the integrated market stays the same but the mark-ups differ between producing for the final good firm in the home or the foreign country as variety producers in addition have to pay trade costs. Depending on the size of these costs it might happen that the mark-up for exports is negative while it is positive for the variety supplied to the domestic market. Hence, for some industries trade costs can prevent single-location sourcing from both final good producers. A specialization pattern might therefore only occur in some industries. A reduction in trade costs would lead to a higher probability of a single variety producer to find itself in a situation of international competition and
would therefore in principle have the same qualitative effects as the scenario of opening to trade without restrictions studied in this paper. One could also assume that due to an implicit home bias effect a firm cannot steal the complete market but only a fixed fraction. This fraction can be changed smoothly to mimic effects of gradual market integration. Second, the model was set up in a simple static framework. Suppose that mark-up and market share shocks only arrive from time to time, say with constant probabilities. The decision making of the firms is hardly affected and openness to trade would again simply imply increased uncertainty about future profitability. Hence, the main results would immediately carry over to such a dynamic setting. Third, I used the term 'market integration' hinting at a merging of two markets into one, which is not entirely the case. In contrast to many trade models the amount of varieties does not double, but stays constant inducing more competition within every variety sector. But there is no reason to rule out that in the long run variety producers can adapt their production techniques and that a persistent specialization pattern evolves. Hence, one should interpret the present model rather in a medium run perspective. Fourth, the model was designed in a very stylized way in order to analyze the effects of openness to trade and employment protection in a qualitative way. For quantitative exercises such as a cross-country welfare evaluation of existing firing restriction legislations given the observed openness to trade, one would have to employ a dynamic version of the model. This is left for future research.

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37 This extension is briefly described in appendix 2.D.
Appendix

2.A Derivations and proofs

Comparative statics w.r.t. F and proof of proposition 2.2.1.

\[ \frac{d\theta}{d\pi^e} > 0, \quad \frac{d\theta}{dM_0} > 0, \quad \frac{d\theta}{dc} < 0, \quad \frac{d\theta}{d\eta} < 0. \] (2.43)

\[ \frac{dy}{dF} = -1, \quad \frac{dy^e}{dy} = \frac{g_Y(y)(y^e - y)}{1 - G} > 0. \] (2.44)

\[ \frac{d\pi^e}{dy} = -(1 - \omega)(1 - G) < 0. \] (2.45)

\[ \frac{d\pi^e}{dF} = -[1 - (1 - \omega)(1 - G)] < 0. \] (2.46)

\[ \frac{dm}{dF} = \frac{1 - \eta}{\eta} \frac{m}{\pi^e} \cdot \frac{d\pi^e}{dF} = \frac{1 - \eta}{\eta} \frac{m^2}{c^\theta} \cdot \frac{d\pi^e}{dF} < 0. \] (2.47)

\[ \frac{dG}{dF} = -g_Y(y) < 0. \] (2.48)

\[ \frac{de}{dF} = (1 - G) \cdot \frac{dm}{dF} - m \cdot \frac{dG}{dF}. \] (2.49)

\[ \frac{de}{dF} = -(1 - G) \frac{1 - \eta}{\eta} \frac{m^2}{c^\theta} [1 - (1 - \omega)(1 - G)] + mg_Y(y) \frac{\chi}{\chi} > 0. \] (2.50)

Proof of proposition 2.2.2. The first part follows directly from the drop in the cut-off value \( y \), according to equation (2.8). For the second part write the marginal change of the average wage \( w^e = (1 - \omega) + \omega(y^e + F) \) as

\[ \frac{dw^e}{dF} = \omega \left( 1 - \frac{g_Y(y)(y^e - y)}{1 - G_Y(y)} \right). \]
Whether the term $\chi$ is smaller or greater than 1 depends on the underlying distribution. For many distributions like uniform, normal, log-normal (with sufficiently small variance), beta (in the range used in this paper) etc. $\chi$ is bound between 0 and 1 for all $y$ which consequently implies that $F$ increases the average wage.

**Proof of lemma 2.3.2.** First, rewrite the MP condition. By the definition of MP one knows that $\int_{-\infty}^{\infty} y dG_Y(y) - \int_{-\infty}^{\infty} y dG_Y(y) = 0$. Integrate by parts to get

$$
\lim_{y \to \infty} \{y [G_Y'(y) - G_Y(y)]\} - \lim_{y \to -\infty} \{y [G_Y'(y) - G_Y(y)]\}
- \int_{-\infty}^{\infty} [G_Y'(y) - G_Y(y)] \, dy = 0.
$$

As both limits tend to zero because of the definition of $G_Y(\cdot)$ and $G_Y'(\cdot)$ as cdfs it can be established that

$$
\int_{-\infty}^{\infty} [G_Y'(y) - G_Y(y)] \, dy = 0. \tag{2.51}
$$

Split the integral such that

$$
\int_{-\infty}^{y} [G_Y'(y) - G_Y(y)] \, dy + \int_{y}^{\infty} [G_Y'(y) - G_Y(y)] \, dy = 0, \quad \forall y \in \mathbb{R}. \tag{2.52}
$$

I will now establish that the first term in (2.52) is non-negative while the second term is non-positive. Consider two cases, first $y \geq \hat{y}$. Integrate the SCS condition from above to get

$$
\int_{y}^{\infty} G_Y(y) \, dy \geq \int_{y}^{\infty} G_Y'(y) \, dy. \tag{2.53}
$$

Note that the same is true in the second case $y \leq \hat{y}$: Integrate the SCS condition from below and use (2.52) to arrive at (2.53) which now consequently holds $\forall y \in \mathbb{R}$. To show that job creation is increasing it suffices to prove that the term $\Lambda \equiv \left[ y' - y \right] (1 - G_Y(y)) = \int_{y}^{\infty} (y - y') dG_Y(y)$ is increasing as a result of a MPSCS. Hence, one has to show that $\Lambda' - \Lambda \equiv \Xi$ is non-negative, i.e.

$$
\Xi = \left[ \int_{y}^{\infty} y dG_Y(y) - \left( 1 - G_Y(y) \right) y \right] - \left[ \int_{y}^{\infty} y dG_Y(y) - \left( 1 - G_Y(y) \right) y \right] \geq 0. \tag{2.54}
$$
Integrate this expression by parts and take the limits to get
\[ \Xi = -y \left[ G_Y'(y) - G_Y(y) \right] - \int_y^\infty \left[ G_Y'(y) - G_Y(y) \right] dy \]
\[ - \left( 1 - G_Y'(y) \right) y + \left( 1 - G_Y(y) \right) y. \]
Terms cancel, leaving \[ \Xi = -\int_y^\infty \left[ G_Y'(y) - G_Y(y) \right] dy \] which, as has been established before, has to be non-negative.

**Proof of corollary 2.3.1.** Recall that in the proof of lemma 2.3.2 it was established that \( \Lambda \equiv (1 - G)(y^e - y) \) is increasing for any kind of MPSCS. Lemma 2.3.1 states that \( G \) is increasing for any kind of SCS if \( y \leq \hat{y} \). Both results can only be true at the same time if \( y^e \) is also increasing. Inserting in the wage equation (2.7) automatically reveals that the average wage \( w^e \) has to rise as well. The increase in the wage dispersion is a direct result of the MPSCS of \( Y \).

**Proof of lemma 2.4.1.** See appendix 2.E for a short summary of the used mathematical concepts that are used in this proof. First, note that \( Q = 2B \), where \( B \) is Bernoulli-distributed with probability parameter \( 1/2 \). Consequently, \( E(Q) = Var(Q) = 1 \). One can therefore establish that \( E(Y) = E(Y') = E(Q)E(\Phi) = E(\Phi) \), hence we have mean preservation. Second, due to independence the variance of \( Y' \) is given as \( Var(Y') = E(Q)^2Var(\Phi) + E(\Phi)^2Var(Q) + Var(Q)Var(\Phi) = 2Var(\Phi) + E(\Phi)^2 \) which is clearly bigger than the variance of \( Y \), i.e. \( Var(Y) = Var(\Phi) \). The density and distributions of \( Y \) and \( Y' \) are given as
\[ g_Y(y) = g_\Phi(y) \quad \text{and} \quad g_{Y'}(y) = g_\Phi \left( \frac{y/2}{4} \right), \] (2.55)
\[ G_Y(y) = G_\Phi(y) \quad \text{and} \quad G_{Y'}(y) = \frac{1 + G_\Phi(y/2)}{2}, \] (2.56)
where \( g_{Y'}(y) \) is derived by using the density formula for product distributions and \( G_{Y'}(y) \) simply stems from integration. Third, I show that \( G_Y(\cdot) \) and \( G_{Y'}(\cdot) \) fulfill the SCS condition. Recall that \( \Phi \) is bounded between 0 and \( \bar{\phi} \). If the cdfs above are evaluated at these boundaries it is clear to see that
\[ G_Y(0) < G_{Y'}(0) \quad \text{and} \quad G_Y(\bar{\phi}) > G_{Y'}(\bar{\phi}). \] (2.57)
Consequently, because of continuity there has to be an intersection point \( \hat{y} \) such that \( 0 < \hat{y} < \bar{\phi} \). It is not clear that this intersection point is unique which is required for the SCS condition. A sufficient condition is that \( g_\Phi(y/2) < 4g_\Phi(y) \). Note that the number of crossings is uneven. If there was more than
one crossing \( \hat{y} \) then at least one of the intersection points would be characterized by \( g_Y(\hat{y}) < g'_Y(\hat{y}) \), i.e. \( G_Y \) intersects \( G_Y' \) from below. Given the condition from above this cannot be true. A stronger condition that is sufficient for uniqueness is that \( g_\Phi(\cdot) \) is non-decreasing.

**Proof of proposition 2.5.1.** The derivative of 2.31 w.r.t. \( F \) reads

\[
\frac{d\Omega(F)}{dF} = \frac{dm}{dF} \int_\hat{y}^\infty y dG_Y(y) + mg(y)(z - F) - \frac{dm}{dF} z + \frac{dm}{dF} G(z - F) - mG - mg(y)(z - F) - c \frac{d\theta}{dF}.
\]

(2.58)

Use \( \frac{dm}{dF} = (1 - \eta)m^f \cdot \frac{d\theta}{dF} \) and rewrite this equation as

\[
\frac{d\Omega(F)}{dF} = \left( m^f \left[ (1 - \eta)(y^e - z + F)(1 - G - F) - c \right] \frac{d\theta}{dF} + m^f \eta F \frac{d\theta}{dF} - mG. \right.
\]

(2.59)

Given that the Hosios-condition holds the first term is zero because of the free entry condition (2.29). Using \( \frac{d\theta}{dF} = -\frac{m}{\eta c} [\omega + (1 - \omega)G] \) leaves

\[
\frac{d\Omega(F)}{dF} = - \frac{m^2}{\theta} \left[ \omega + (1 - \omega)G \right] \frac{F}{c} - mG < 0.
\]

(2.60)

**Proof of proposition 2.5.2.** Note that the welfare function (2.31) can be rewritten as

\[
\Omega(F) = e\omega [y^e - z + F] + z = \omega m\Lambda + z
\]

(2.61)

as explained in appendix 2.B. Given the assumptions, \( \Lambda \) and \( m \) are both non-decreasing as a result of a MPSCS as established in the proof of lemma 2.3.2.

**Proof of proposition 2.5.3.** It has to be established under which conditions (2.60) is decreasing in job turnover, i.e. \( m \) and \( G \). The condition \( \eta \leq 1/2 \) guarantees that the term \( \frac{m^2}{\theta} \) in (2.60) is increasing in \( \theta \). In that case \( \frac{d\Omega(F)}{dF} \) is decreasing for any increase in \( m \) and/or \( G \).

**Proof of proposition 2.5.4.** First note that the budget solving lump-sum tax rate (2.42) changes with \( F \) according to

\[
\frac{dT}{dF} = -d \frac{de}{dF} - \frac{dm}{dF} GF\psi - \frac{dG}{dF} mF\psi - mG\psi.
\]

(2.62)
Let me define $\bar{w} = w(y) - T$ and $\bar{z} = z - T$. Use the envelope theorem, insert
for $\frac{\partial m}{\partial F} = \frac{\partial e}{\partial F} \frac{1}{1-G} - mg(y) \frac{1}{1-G}$, expand and rewrite the expression as

$$
\frac{d\Omega(F)}{dF} \bigg|_{F=0} = \frac{de}{dF} \cdot [u(\bar{w}) - u(\bar{z})] + \frac{de}{dF} \cdot b [eu'(\bar{w})e + (1-e)u'(\bar{z})]
- mg(y) [u(\bar{w}) - u(\bar{z})] + ewu'(\bar{w})e + (1-e)u'(\bar{z})mG\psi,
$$
(2.63)

where superscript $e$ again indicates the conditional expectation. All terms in brackets are positive. The first is so because $w(y) \geq z, \forall y \geq y$. The condition stated in proposition 2.5.4 can then be directly derived from (2.63).

### 2.B Utility and output maximization

Both concepts, utility and output maximization, coincide in case of risk-neutral workers. Consider the second best welfare function for utility maximization

$$
\Omega^{util} = m \int_{y}^{\infty} w(y) dG(y) + (1-e)z, \tag{2.64}
$$

subject to the equilibrium conditions. Insert the wage schedule (2.5) and rearrange to get

$$
\Omega^{util} = e(1-\omega)z + e\omega ye + e\omega F + (1-e)z. \tag{2.65}
$$

Rewrite the free entry condition (2.29) as $e(1-\omega) [y - z + F] - mF - c\theta = 0$ and add it to (2.65) to get

$$
\Omega^{util} = eye + (1-e)z - mGF - c\theta = \Omega^{output}, \tag{2.66}
$$

which is identical to the output maximizing objective function (2.31).

### 2.C Goods markets clearing

This section briefly describes goods markets clearing in the closed economy. The intermediate goods markets are completely supply-driven and therefore clear trivially. Supply of an intermediate good is either 1 or 0 depending on whether or not a worker is kept. The demand correspondence of the final good firm for every variety is $[0,1]$, hence all intermediate goods markets clear. I now address the final good market. The final good production function states that given that $e$ intermediate goods firms survive also the amount of the final good output is $e$. In addition, an unemployed person produces $h$ units in terms
of the final good as home production. Hence, equating supply and demand gives

\[ e + (1 - e)h = ew^e + (1 - e)h + mGF + c\theta + e(1 - \phi^e), \]

where \( w^e = \frac{\int_y^y w(y) dG_Y(y)}{1 - G_Y(y)} \) and \( \phi^e = \frac{\int_{\phi^e}^{\phi} \phi dG_\Phi(\phi)}{1 - G_\Phi(\phi)} \).

These denote the average wage and the average mark-up which enter because \( 1 - \phi^e \) in (2.67) are marginal costs for the usage of the capital good. The right hand side of equation (2.67) describes demand for the final good which consists of consumption of the employed workers using their wage income, consumption of the home production of unemployed workers, and all costs of the intermediate goods firms for firing, vacancy posting and production. Insert the wage schedule (2.5) to get

\[ e = e(1 - \omega)h + e\omega y^e + e\omega F + mGF + c\theta + e - e\phi^e. \] (2.68)

Use the free entry condition (2.29) to insert for \( c\theta = e(1 - \omega) [y^e - h] - mGF - e\omega F \) to arrive at

\[ e = e y^e + e - e\phi^e \quad \Leftrightarrow \quad y^e = \phi^e. \] (2.69)

As \( q = 1 \) is true for all producing matches it follows that \( y = \phi \) above the cut-off and hence \( y^e = \phi^e \) has to hold which completes the proof.

### 2.D Intermediate market shares

Suppose that the result of the contest is not a ‘winner-takes-all’ solution. Instead suppose that

\[ q = \begin{cases} 2(1 - s) & \text{with probability } \frac{1}{2}, \\ 2s & \text{with probability } \frac{1}{2}, \end{cases} \] (2.70)

where \( s \in (1/2, 1] \). \( s = 1 \) represents the extreme case from before, while \( s \to 1/2 \) mimics the closed economy case where every firm just supplies half of the world market. Hence, before-wage profits \( Y \) are given by \( Y = Q \cdot \Phi \), where \( Q = 2[s + (1 - 2s)B] \) and \( B \) is again Bernoulli-distributed with probability parameter \( 1/2 \). Clearly, \( E(Q) = 1 \) and \( Var(Q) = (1 - 2s)^2 \). It is easy to see that

\(^{38}\text{UI b is set to 0 in what follows. If UI is financed through taxes as in section (2.5.3) the inclusion is trivial as it just implies a redistribution of final goods but does not change the market clearing condition.}\)
$E(Y) = \mu_\Phi$ and $\text{Var}(Y) = \sigma^2_\Phi + \mu^2_\Phi (1 - 2s)^2 + \sigma^2_\Phi (1 - 2s)^2$. Hence, the variance increases in $s$ while the expectation is unchanged. Density and distributions are given by

$$g_Y(z) = \frac{1}{2} \left[ \frac{1}{2s} \Phi \left( \frac{z}{2s} \right) + \frac{1}{2(1-s)} \Phi \left( \frac{z}{2(1-s)} \right) \right], \quad (2.71)$$

$$G_Y(z) = \frac{1}{2} \left[ G_\Phi \left( \frac{z}{2s} \right) + G_\Phi \left( \frac{z}{2(1-s)} \right) \right]. \quad (2.72)$$

Observe how the closed and the open economy setting described in the paper are nested as limiting cases $s \to 1/2$ and $s = 1$. The parameter $s$ therefore captures the strength of the effect of market integration on the revenue risk.

2.6 Math sheet

This section summarizes general mathematical concepts that have been heavily used in this paper for easier reference.

Change of variables

Let $X$ be a univariate continuous random variable with cdf $G_X(\cdot)$ and pdf $g_X(\cdot)$. Let $f$ be a monotone transformation such that $Y = f(X)$. Then the cdf and pdf of $Y$ are defined as follows

a) if $f(\cdot)$ is increasing

$$G_Y(y) = G_X(f^{-1}(y)) \quad \text{and} \quad g_Y(y) = g_X(f^{-1}(y)) \frac{1}{f'(f^{-1}(y))},$$

b) if $f(\cdot)$ is decreasing

$$G_Y(y) = 1 - G_X(f^{-1}(y)) \quad \text{and} \quad g_Y(y) = -g_X(f^{-1}(y)) \frac{1}{f'(f^{-1}(y))}.$$

Random variables algebra

Let $X$ and $Y$ be two independent univariate continuous random variables with the according cdfs and pdfs. Define $Z = X + Y$. Then $G_Z(\cdot)$ is the convolution

$$G_Z(x) = (G_X * G_Y)(x) = \int_{-\infty}^{\infty} G_X(x - y) dG_Y(y) = \int_{-\infty}^{\infty} G_Y(x - y) dG_X(y),$$

$$g_Z(x) = (g_X * g_Y)(x) = \int_{-\infty}^{\infty} g_X(x - y) g_Y(y) dy = \int_{-\infty}^{\infty} g_Y(x - y) g_X(y) dy.$$
Now define $Z = X - Y$. Then $G_Z(\cdot)$ is the cross-convolution

$$G_Z(x) = (G_X \ast G_Y)(x) = \int_{-\infty}^{\infty} [1 - G_Y(y - x)] \, dG_X(y) = \int_{-\infty}^{\infty} G_X(x + y) \, dG_Y(y),$$

$$g_Z(x) = (g_X \ast g_Y)(x) = \int_{-\infty}^{\infty} g_Y(y - x)g_X(y) \, dy = \int_{-\infty}^{\infty} g_X(x + y)g_Y(y) \, dy.$$ 

All values were assumed to be real-valued. Now define $Z = X \cdot Y$. $g_Z(\cdot)$ and $G_Z(\cdot)$ are then given by

$$g_Z(z) = \int_{-\infty}^{\infty} \frac{1}{|x|} g_X(x) \, g_Y(z/x) \, dx,$$

$$G_Z(z) = \int_{-\infty}^{z} g_Z(x) \, dx.$$ 

Expectation and variance can be computed as

$$E(Z) = E(Z)E(X),$$

$$Var(Z) = E(X)^2 Var(Y) + E(Y)^2 Var(X) + Var(X) Var(Y).$$
### 2.F Tables and figures

#### Table 2.1: Numerical example

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u(x) = \frac{x^{1-\psi} - 1}{1-\psi}$</td>
<td>$\theta = 2.81$</td>
</tr>
<tr>
<td>$m = M_0\theta^{1-\eta}$</td>
<td>$\gamma = 0.15$</td>
</tr>
<tr>
<td>$Y \sim \text{beta}(\alpha, \beta)$</td>
<td>$\bar{e} = 0.9$</td>
</tr>
<tr>
<td>$b = 0.1$</td>
<td>$m = 0.96$</td>
</tr>
<tr>
<td>$F = 0$</td>
<td>$m^f = 0.34$</td>
</tr>
<tr>
<td>$\psi = 3$</td>
<td>$G = 0.06$</td>
</tr>
<tr>
<td>$\eta = 0.5$</td>
<td>$\frac{e}{F} = 0.3$</td>
</tr>
<tr>
<td>$\omega = 0.5$</td>
<td>$T = 0.01$</td>
</tr>
<tr>
<td>$\psi = 1$</td>
<td>$\Gamma = 16.72$</td>
</tr>
<tr>
<td>$\alpha = 2$</td>
<td>$\frac{d\Gamma}{F} = -1.97$</td>
</tr>
<tr>
<td>$\beta = 2$</td>
<td>$\frac{d\Gamma}{m} = -2.87$</td>
</tr>
<tr>
<td>$h = 0.05$</td>
<td>$\frac{d\Gamma}{c} = -0.77$</td>
</tr>
<tr>
<td>$c = 0.06$</td>
<td>$\frac{d\Gamma}{\alpha} = 0.14$</td>
</tr>
<tr>
<td>$M_0 = 0.57$</td>
<td>$\frac{d\Gamma}{\alpha} = -28.33$</td>
</tr>
</tbody>
</table>

#### Figure 2.1: Cumulative distribution functions - mean preserving single crossing spread
Figure 2.2: Probability density functions - mean preserving single crossing spread
Chapter 3

Cyclical Unemployment Benefits and Non-Constant Returns to Matching

Philip Schuster†

†The paper was presented at the University of St. Gallen and the annual meeting of the Austrian Economic Association (2012, Vienna). I thank the participants for helpful comments and discussions. In particular I thank Gabriel Felbermayr, Reto Föllmi, Christian Keuschnigg and Jochen Mankart for their advice. All remaining errors are my own.


3.1 Introduction

Ever since the seminal work by Baily (1978) the literature has studied questions of optimal unemployment benefits (UB) in frameworks that allow a trade-off between reduced incentives to search and gains from insurance. While many extensions and related issues\(^1\) like optimal duration dependence (e.g. Hopenhayn and Nicolini, 1997 and Shimer and Werning, 2008) or liquidity concerns (Chetty, 2008) have been addressed, surprisingly the question of how to set UB optimally over the business cycle has only very recently sparked the interest of economists. Moyen and Stähler (2009) and Andersen and Svarer (2010) show that UB should be counter-cyclical if the government faces an intertemporal budget constraint in order to help workers to smooth consumption over the cycle. Many papers like Kiley (2003), Sánchez (2008), Kroft and Notowidigdo (2011) and Schmieder et al. (2011) emphasize a different argument. UB generosity should be counter-cyclical if the disincentive problem is less severe in downturns. Landais et al. (2010) provide a theoretical reason for such a mechanism. They also derive counter-cyclical optimal UB by arguing that the negative within-group search externality is stronger in bad times, i.e. the own search is more harmful to other searchers if the market is tight or jobs are rationed. This implies that from a social point of view, there is too much search in bad times and UB should be more generous in recessions. This search correcting function of UB will be refer to as the Pigouvian role. This additional congestion effect is not present in the canonical Diamond-Mortensen-Pissarides (DMP) model. Landais et al. (2010) introduce it by assuming decreasing marginal product of labor and sticky wages which generates ‘job rationing’\(^2\) and excessive search in bad times. While the principle economic intuition is very appealing this framework might be subject to the ‘Pissarides (2009)-critique’: Although a high degree of wage stickiness is observed during employment spells, ‘outside’ wages for new employees, which matter for job creation, vary almost one for one with productivity as given by standard Nash bargaining.

Without running afoul of the ‘Pissarides-critique’, this paper provides an alternative theoretical motivation for state dependent variation in the extend of search externalities. This is achieved by allowing for non-constant returns to scale in matching. Hence, in bad times the labor market does not only experience lower productivity but also more (or less) congestion than is socially

\(^1\)See Fredriksson and Holmlund (2006) for a survey.

\(^2\)See Michaillat (2012) for details.
optimal due to changes in employment levels depending on whether returns to scale are decreasing (or increasing). This change in congestion is symmetric for both workers and firms in contrast to the typical search externalities that are negative within the groups of workers or firms but positive between them. Keller et al. (2010) established that efficiency can be restored by an employment tax (subsidy) if returns are decreasing (increasing). This is shown by resorting to an abstract proportional match surplus tax/subsidy. I will show that such a tax/subsidy is not at the disposal of a policy maker as it cannot be mimicked by any proportional taxation or subsidization of economic flows. Hence, any efficient policy has to be a function of the match surplus and therefore of the business cycle. I establish that UB have to be set pro-cyclically (countercyclically) if the matching function exhibits decreasing (increasing) returns to scale even if workers are risk-neutral which isolates the Pigouvian role of UB from their function of insurance provision.

The existing literature almost exclusively used constant returns to matching. Typical arguments brought up in favor of using constant returns to scale specifications are: analytic simplicity, consistency with a balanced growth path and empirical support. I will shortly comment on all these points. Using constant returns typically reduces the dimensionality of the matching process by one and makes equilibrium recursive and easily tractable. This is a fair argument and its validity obviously depends on the reduction in complexity one is willing to make and that seems to be appropriate for a given problem. The argument that only constant returns to scale are consistent with a balanced growth path cannot be supported unreservedly. The argument is certainly valid with respect to population or labor force growth that will lead to different growth rates of total output depending on the returns to scale assumption. However, non-constant returns to scale matching functions, as presented in this paper, are perfectly in line with a balanced growth path concerning exogenous technology growth. I will now turn to the question of empirical evidence. Probably the most comprehensive treatment is the meta study by Petrongolo and Pis-

---

3Except for employment no other stocks are present in the model.
4Some authors like Marimon and Zilibotti (1999), Acemoglu and Shimer (2000) and Acemoglu (2001) stress an additional function of UB, also independent of the insurance provision argument. UB gives workers the time to look for more suitable jobs which increases productivity. As jobs are homogeneous in my model this function will not play a role.
5Notable exceptions are Diamond (1982), Pissarides (1984), Howitt and McAfee (1987) and Hyde (1997). Keller et al. (2010) provide a comprehensive summary of these papers.
6Another exhaustive summary of empirical results concerning the matching function is presented by Broersma and van Ours (1999).
sarides (2001) which is often used as a reference to justify constant returns to scale. Table 3.1 reproduces the unrestricted estimates for the matching elasticity of unemployment ($L - N$) and vacancies ($V$) for the studies mentioned in Petrongolo and Pissarides (2001) that explicitly test for constant returns to scale. By inspection of this table it seems hard to claim that there is strong evidence for constant returns to scale of the matching function. Anderson and Burgess (2000) for the United States, Layard et al. (1991), Pissarides (1986) and Coles and Smith (1996) for the United Kingdom, and van Ours (1991) for the Netherlands support constant returns to scale. In contrast Warren (1996) and Blanchard and Diamond (1991) for the United States, Yashiv (2000) for Israel, Kangasharju et al. (2005) for Finland and Münich et al. (1999) for the Czech Republic find increasing returns. Decreasing returns are supported by Burda and Wyplosz (1994) for France, Germany, Spain and the United Kingdom, by Berman (1997) for Israel, by Burgess and Profit (2001) for the United Kingdom and by Fahr and Sunde (2004) for Germany.

### Table 3.1: Empirical evidence on matching elasticities

<table>
<thead>
<tr>
<th>Sample</th>
<th>ln($L - N$)</th>
<th>ln($V$)</th>
<th>crts test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pissarides (1986)</td>
<td>-</td>
<td>-</td>
<td>✓</td>
</tr>
<tr>
<td>Layard et al. (1991)</td>
<td>-</td>
<td>-</td>
<td>✓</td>
</tr>
<tr>
<td>Bergman (1997)</td>
<td>0.29</td>
<td>0.39</td>
<td>×</td>
</tr>
<tr>
<td>Burda and Wyplosz (1993)</td>
<td>0.52</td>
<td>0.09</td>
<td>×</td>
</tr>
<tr>
<td>Germany</td>
<td>0.68</td>
<td>0.27</td>
<td>×</td>
</tr>
<tr>
<td>Spain</td>
<td>0.12</td>
<td>0.14</td>
<td>×</td>
</tr>
<tr>
<td>UK</td>
<td>0.67</td>
<td>0.22</td>
<td>×</td>
</tr>
<tr>
<td>Yashiv (2000)</td>
<td>0.49</td>
<td>0.87</td>
<td>×</td>
</tr>
<tr>
<td>Warren (1996)</td>
<td>sum 1.33</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Anderson and Burgees (2000)</td>
<td>0.43</td>
<td>0.81</td>
<td>✓</td>
</tr>
<tr>
<td>USA: All new hires</td>
<td>0.39</td>
<td>0.75</td>
<td>✓</td>
</tr>
<tr>
<td>USA: previously employed</td>
<td>0.54</td>
<td>0.87</td>
<td>✓</td>
</tr>
</tbody>
</table>

Note: Reproduced from Petrongolo and Pissarides (2001). Only studies with tests for constant returns to scale (crts) are included. The tests are reported at a 10%-level. $L - N$ and $V$ denote number of unemployed and vacancies.

Fahr and Sunde (2004) further estimate matching elasticities for different occupations and find substantial heterogeneity at this disaggregate level. Crafts and technical occupations seem to exhibit increasing returns while industrial, white collar and social occupations are related with decreasing returns. This result suggests that policy implications might vary considerably between groups of occupations. Broersma and van Ours (1999) and Sunde (2007) argue that estimates concerning the returns to scale can be severely biased by neglecting unobserved search intensity and on-the-job search. While former can be
corrected relatively easy by making careful distinctions between ‘conditional’ and ‘unconditional’ matching functions\(^7\) when relating empirical estimates to the theoretical model, correcting for the latter is less trivial and will not be attempted in this paper.

The aggregate matching function is usually used as a ‘black box’ tool. There have been several attempts of microfounding the matching function. None of them suggests a regularity such that the underlying frictions can only result in a constant returns to scale specification. One of the first microfoundations is due to Butters (1977) and Hall (1979) and reflects a coordination failure by mimicking the problem of randomly placing balls in urns. They derive the following matching function \(M = V \left[1 - (1 - 1/V)^{L-N}\right]\) which has decreasing returns and only approximately converges to a constant returns to scale specification for large \(V\), namely \(M = V(1 - e^{-(L-N)/V})\). In an extension, Calvó-Armengol and Zenou (2005) microfound matching in a framework where workers are embedded in a social network and can find vacancies also through word-of-mouth. They show that the network size plays a crucial role for the efficiency of their matching process. On one hand, if the network size increases the coordination failure shrinks because unemployed workers potentially learn about more job opportunities. On the other hand, coordination failure increases as it becomes more likely that a single worker receives several job offers at the same time. First, the positive effect dominates, then the negative effect is more important. Hence, for small networks the returns to scale are increasing while they are decreasing for oversaturated networks. Other microfoundations focus on mismatch of the form that workers and vacancies are randomly assigned to submarkets \(\ell = 1, 2, \ldots\) that clear, i.e. \(M_{i\ell} = \min\{L_{\ell} - N_{\ell}, V_{\ell}\}\), but where workers and vacancies are immobile and cannot (or only slowly) move to another submarket once they found themselves on the long side of their current submarket. This type of friction has been discussed in the literature since the 1970s, see for example Hansen (1970). More recent papers are provided by Lagos (2000) and Shimer (2005b). Latter derives an aggregate matching function that is increasing in market size, i.e. the measures of workers and vacancies in each submarket, although in a simulation exercise the matching function is virtually indistinguishable from a Cobb-Douglas specification. Stevens (2007) models the underlying friction as an explicit time consuming process of searching and evaluating potential matches, as they are heterogeneous, in form of a telephone-line-queuing model. She derives the following aggregate matching

\(^7\)I use the terminology as in Stevens (2007).
function \( M = P_{\text{accept}} \cdot \frac{(L-N)V}{L-N+V} \), where \( P_{\text{accept}} \) is the probability that a job offer is accepted. Observe that the second term is homogeneous of degree one, but the matching function shares this property only if \( P_{\text{accept}} \) is independent of \( L - N \) and \( V \) in levels. It is easy to generate a scenario where this is not the case, e.g. the existence of a simple welfare state where employed workers pay taxes and unemployed workers receive tax-financed benefits introduces level-dependence in job acceptance rates.

To summarize, first, there are no fundamental reasons derived from explicit microfoundation of why the matching function should exhibit constant returns to scale. Second, as the empirical evidence concerning the returns to scale of the matching process seems to be rather inconclusive and mixed with a high degree of country, regional and occupation specific heterogeneity, it seems reasonable to allow for a more flexible specification of the matching function in the theoretical model, as I will do in this paper.

The paper further relates to the strand of the literature that - in contrast to Chetty (2006a) - has focused on the insurance efficiency trade-off in general equilibrium settings where job finding probabilities are influenced by firms’ behavior through vacancy creation and wage bargaining. Cahuc and Lehmann (2000), Fredriksson and Holmlund (2001), Coles and Masters (2006), Lehmann and van der Linden (2007), and Coles (2008) look at optimal policy in steady state while Mitman and Rabinovich (2011) also take business cycle dynamics into account. Jung and Kuester (2011) extend their analysis by looking at a broader set of instruments and find that all, a recruitment subsidy, a firing tax and unemployment benefits have to rise in recession. However, none of those papers deals with the Pigouvian role of UB to correct for inefficient levels of aggregate search intensity as in this paper.

Probably the closest related study is the paper by Keller et al. (2010). They also introduce a flexible matching function nesting different returns to scale specifications and derive the mentioned efficiency results. However, the main focus of their paper is not the implementation of an optimal allocation but rather market size effects introduced through non-constant returns to scale when labor force participation is endogenous. They further emphasize the different dynamic behavior of job finding rates in their model in contrast to the canonical

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8I normalized search costs to 1 without loss of generality for my argumentation.
DMP model where job finding probabilities directly jump to the new equilibrium values after a shock.

The paper is organized as follows. Section 3.2 extends the canonical DMP model such that it additionally allows for non-constant returns to scale in matching, decreasing returns to labor in production, and sticky wages. It can therefore nest different combinations of assumptions. Section 3.3 derives equilibrium while section 3.4 characterizes the social optimal allocation and possible implementations. Section 3.5 adds risk-aversion to analyze the interaction of the insurance and the externality correction purpose of UB. The section closes with a small numerical illustration. Section 3.6 concludes.

3.2 Model description

The model is set up in discrete time and I will focus exclusively on its comparative static behavior at the steady state. All values are denoted at end of period. The model is based on the simple, canonical DMP model as presented in Pissarides (2000).

3.2.1 Matching technology

In contrast to the textbook model I allow for the following general form of the matching function similar to Keller et al. (2010). The number of total matches \( M \) is

\[
M = \Phi(m(s(L - N), V)) = \Phi(m(L \cdot su, L \cdot v)),
\]

where \( L \) denotes exogenously fixed labor force, \( V \) is the number of vacancies, \( N \) is the number of employees, such that \( L - N \) denotes the number of unemployed. \( u \) is the unemployment rate, \( v \) is the vacancy rate and \( s \) denotes average search effort. I interpret \( m \) as amount of aggregate search activity.\(^{11} \) \( m(\cdot, \cdot) \) is assumed to be increasing in both arguments and homogeneous of degree one. Because of that one can rewrite \( M = \Phi(Lu \cdot m(s, \theta)) \), where \( \theta \) denotes labor market tightness of form \( \theta \equiv \frac{V}{L-N} = \frac{v}{u} \). The elasticity\(^{12} \) of \( m \) w.r.t. \( u \) and \( v \) is
denoted $\epsilon_u^m \equiv \eta$ and $\epsilon_v^m \equiv 1 - \eta$, respectively, and assumed to be a constant.

The function $\Phi(x)$ for some $x > 0$ is strictly increasing and has a constant elasticity denoted by $\epsilon_\Phi x \equiv \xi$. If $\xi = 1$ one is back in the canonical model with constant returns to scale. $\xi < (>) 1$ implies decreasing (increasing) returns.

Consequently, the elasticities of $M$ w.r.t. $u$ and $v$ are $\epsilon_u^M = \eta \cdot \xi \equiv \eta_u$ and $\epsilon_v^M = (1 - \eta) \cdot \xi \equiv \eta_v$, respectively. Although, I allow for increasing returns to scale I rule out increasing returns to every single factor, i.e. $\eta_u, \eta_v \in (0, 1)$. Hence, $M$ is concave in both $u$ and $v$. The following further conditions have to be fulfilled by $M$ by assumption: $M \leq \min \{L - N, V\}$, $M = 0$ if either $L = N$ or $V = 0$ and $L >> 1$ such that $L \cdot m(su, v) > 1$. Define the following

$$q^w(s_i, s, u, \theta) = s_i \frac{M}{Lsu}, \quad (3.2)$$

as the matching probability for worker $i$ where the matching probability per unit of search $\frac{M}{Lsu}$ is multiplied by the worker’s individual search effort $s_i$. As workers are homogeneous they will always pick the same search effort in equilibrium, i.e. $s_i = s$, in which case every worker faces the following probability

$$q^w(s, u, \theta) = \frac{\Phi(Lu \cdot m(s, \theta))}{Lu}, \quad (3.3)$$

while the matching probability of a firm is simply $q^f(s, u, \theta) \equiv q^w(s, u, \theta)$. Note that in this general formulation $q^w$ can also depend on the level of unemployment not only relative unemployment reflected by the labor market tightness like in the canonical model. Four important derived elasticities$^{14}$ that will be used extensively are

$$\epsilon^q_{s}^w |_{s_i = s} = 1 > 0, \quad \epsilon^q_{\theta} = \eta_u > 0, \quad \epsilon^q_{\theta} = \eta_v > 0, \quad \epsilon^q_{u} = \xi - 1. \quad (3.4)$$

Note that the sign of $\epsilon^q_{u}^w$ depends on our returns to scale assumption. If returns to scale are increasing, i.e. $\xi > 1$, then there is an additional positive level effect of the number of searches on the efficiency of matching. In a business cycle context this would imply that while the direct effect of lower factor productivity clearly increases unemployment the effect is dampened by the fact that for a higher number of searchers matching becomes more effective. On the other hand if $\xi < 1$ the direct effect is enforced as matching becomes less effective if

$^{13}$This is supported by empirical evidence, see Petrongolo and Pissarides (2001), and furthermore has the important theoretical implication that the negative within group congestion externalities are still present.

$^{14}$See the appendix for the derivations.
$u$ is high. The job finding rate reacts as follows to the degree of returns to scale and the market size

$$
\epsilon^q_{\xi} = \xi \cdot \ln(L \cdot m(su, v)) > 0, \quad \epsilon^q_{\xi} = \xi - 1. \tag{3.5}
$$

Clearly, unless $\xi \neq 1$ the job finding rate is influenced by the market size. Absolute employment $N$ evolves as follows

$$
N' = M + (1 - \pi^x)N. \tag{3.6}
$$

Next period’s employment $N'$ is simply the current stock minus separations plus the number of new matches. From a firm’s perspective\textsuperscript{15} this law of motion can be rewritten as

$$
N' = q^f V + (1 - \pi^x)N. \tag{3.7}
$$

From the perspective of the workers the law is written as $N' = q^w(L - N) + (1 - \pi^x)N$. Evaluating in steady state gives the standard Beveridge curve,

$$
N = L(1 - u) = \frac{q^w}{\pi^x + q^w} L, \quad L - N = Lu = \frac{\pi^x}{\pi^x + q^w} L. \tag{3.8}
$$

### 3.2.2 Representative firm

There is a single, competitive, representative firm with the following production function

$$
Y = aF(N) \quad \text{with} \quad F(N) = N^\alpha, \quad 0 < \alpha \leq 1. \tag{3.9}
$$

Hence, the production function nests constant as well as decreasing\textsuperscript{16} marginal product of labor. The parameter $a$ is interpreted as an exogenous productivity shift parameter that will capture the state of the economy along the business cycle and will later be used for comparative static exercises. Although production is handled by a single firm, the firm is assumed to be naive in the following respects to capture important characteristics of a competitive market with a large number of participants.

**Assumption 3.2.1. Naive representative firm:** (a) The firm takes the matching probability $q^f$ as given. (b) The firm assumes $\frac{\partial w}{\partial N} = 0$.

\textsuperscript{15}As the representative firm will take matching probabilities as given it has to maximize profits subject to (3.7) instead of (3.6). See section 3.2.2.

\textsuperscript{16}A typical justification is that the capital stock is fixed or adjusts only sluggishly in the short run. Hence, labor reallocates faster than capital.
Assumption 3.2.1(a) implies that the firm does not internalize the typical search externalities that are present in the canonical framework. Assumption 3.2.1(b) states that the firm does not realize that it can influence the wage (if possible) by deciding how many vacancies to post. This is not an issue if wages are exogenous, or if they are bargained but marginal returns to labor are constant, i.e. $\alpha = 1$. For the case of bargained wages and decreasing marginal returns to labor, wages depend on $N$ via the marginal product. Stole and Zwiebel (1996) showed that in a setting where the firm treats every worker as being marginal during the bargaining it would strategically over-hire to compress wages.\(^\text{17}\) I abstract from this strategic component by assuming the firm to be naive in that respect. The firm’s discounted profits for a given number of workers $N$ can be recursively written as

$$
\Pi(N) = \max_V \left[ aF(N) - wN - cV + \Pi(N') \right] \frac{1}{1+r}, \text{ s.t. (3.7)}. \quad (3.10)
$$

Define $J \equiv \Pi'(N)$ as the marginal value of an additionally filled position for the firm. The optimality condition reads

$$
-c + J q^f = 0 \quad \Rightarrow \quad J = \frac{c\theta}{q^w}. \quad (3.11)
$$

The envelope condition implies that

$$
J = \left[ aF'(N) - w - \frac{N\partial w}{\partial N} + J(1 - \pi^x) \right] \frac{1}{1+r}. \quad (3.12)
$$

By assumption 3.2.1(b) one can rewrite the envelope condition as

$$
J = \frac{y - w}{r + \pi^x}, \quad (3.13)
$$

where I defined output of a marginal worker as $y \equiv aF'(N) = aF'(L(1-u))$.\(^\text{18}\) Combining the optimality and the envelope condition gives the typical job creation (JC) condition that equates benefits and costs of marginally increasing $N$, i.e.

$$
\frac{y - w}{r + \pi^x} = \frac{c\theta}{q^w}. \quad (3.14)
$$

\(^\text{17}\)Cahuc and Wasmer (2001) show that this result disappears if the production function has decreasing returns in both, labor and capital, but constant returns to scale.

\(^\text{18}\)Clearly, in the case of constant returns to labor, $\alpha = 1$, marginal output is equal to the exogenous productivity parameter, i.e. $y = a$. 
3.2.3 Workers

Workers are assumed to be risk-neutral such that one can disentangle the welfare effect of UB because of ‘search correction’ in contrast to insurance.\(^{19}\) A worker \(i\) can be in two discrete states: employed or unemployed. The corresponding values are denoted \(W_i\) and \(U_i\),

\[
U_i = \max_{s_i} \left[ z_i + q_i^w W_i + (1 - q_i^w)U_i \right] \frac{1}{1 + r}, \tag{3.15}
\]

where the instantaneous value of unemployment \(z_i \equiv h + b - k(s_i)\) is home production plus UB minus search effort in monetary terms. The effort function has the usual properties, i.e. \(k'(\cdot) > 0\) and \(k''(\cdot) > 0\). The value of being employed is given by

\[
W_i = [w_i - T + \pi^x U_i + (1 - \pi^x)W_i] \frac{1}{1 + r}. \tag{3.16}
\]

Here \(T\) denotes a tax on employed workers which will finance UB for the unemployed. Maximization over search effort while taking all market variables as given implies the following first-order condition for optimal search

\[
\frac{\partial q_i^w}{\partial s_i} (W_i - U_i) = k'(s_i), \tag{3.17}
\]

which equates marginal benefits and costs of an additional unit of individual search effort. As mentioned before all workers are identical which implies that in equilibrium all workers choose the same search intensity, hence

\[
W - U = \frac{sk'(s)}{q^w}. \tag{3.18}
\]

Combine (3.15), (3.16) and (3.18) to get

\[
W - U = \frac{w - T - z - sk'(s)}{r + \pi^x}. \tag{3.19}
\]

Inserting this expression in (3.18) gives the typical job search (JS) condition which reveals that search effort increases with the difference of wage and unemployment income

\[
\frac{w - T - z - sk'(s)}{r + \pi^x} = \frac{sk'(s)}{q^w}. \tag{3.20}
\]

\(^{19}\)This assumption is relaxed in section 3.5.
3.2.4 Wage determination

I consider two wage determination rules. First, like in Landais et al. (2010) I allow for simple sticky wages of form

\[
\text{sticky wage: } w = w_0 a^\gamma, \quad 0 < \gamma < 1. \tag{3.21}
\]

In this case the elasticity of the wage w.r.t. productivity, \( \epsilon^w = \gamma \), is less than 1. Second, I allow for bargained wages subject to the following sharing rule\(^{20}\)

\[
W - U = \frac{\omega}{1 - \omega} J. \tag{3.22}
\]

Inserting (3.13) and (3.19) in (3.22) and solving gives the following wage schedule

\[
\text{flexible wage: } w = (1 - \omega) \left[ z + T + sk'(s) \right] + \omega y, \tag{3.23}
\]

with an elasticity of the wage w.r.t. productivity of \( \epsilon^w \approx 0.98 \) for a reasonable calibration\(^{21}\).

3.2.5 Surplus taxation

This short section explains how a proportional surplus tax \( t \) is introduced to the model for the regime of flexible wages. This tax is admittedly rather abstract but it will help in understanding the principle role optimal policy will have to play. Observe that in the case of no such tax the surplus is \( S \equiv W - U + J \). Simply add (3.13) to (3.19) to arrive at\(^{22}\)

\[
S = \left[ \frac{y - T - z - sk'(s)}{r + \pi^x} \right]. \tag{3.24}
\]

Now a surplus tax is introduced such that only \((1-t)S\) is shared among the parties, i.e. \((1-t)S \equiv W - U + J\). Hence, although the sharing rule (3.22) is still valid the values \( J \) and \( W - U \) are now given by

\[
J = (1 - \omega)(1 - t)S \equiv \omega^f S, \quad W - U = \omega(1 - t)S \equiv \omega^w S, \tag{3.25}
\]

where \( \omega^f \) and \( \omega^w \) are the effective bargaining powers for firm and worker. Hence, optimal job creation and optimal job search effort are characterized

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\(^{20}\)For risk-neutral workers this coincides with the first-order condition of a typical Nash bargaining problem, i.e. \( w = \operatorname{argmax}(W - U)^{\omega f^{1-\omega}} \).

\(^{21}\)This value is stated in Pissarides (2009) and confirmed in the numerical section of this paper.

\(^{22}\)Note that if one assumed that \( k(\cdot) \) was of simply iso-elastic form with constant elasticity \( \epsilon^k = \nu > 1 \) one could rewrite the term \( k(s) - sk'(s) \) as \((1-\nu)k(s)\). Consequently, \( z + sk'(s) = b + h + (\nu - 1)k(s)\).
by the following two conditions

\[ \omega^S f S = \frac{c \theta q^w}{q^w}, \quad \omega^w S = \frac{sk'(s)}{q^w}. \] (3.26)

The next section will describe this in more detail.

### 3.3 Equilibrium

In addition to the derived conditions also the government’s budget has to be balanced, therefore the following has to hold

\[ NT = (L - N)b \quad \Leftrightarrow \quad T = \frac{u}{1 - u}b. \] (3.27)

One can now simply eliminate \( T \) in all corresponding equations. Further, observe that I excluded the surplus tax from the government’s budget, i.e. I assume it to be uncompensated. This has the following reason. In section 3.4 I argue that such a tax is not at the disposal of the policy maker and cannot be mimicked by a set of instruments (even if they are uncompensated as well). The tax will just help to characterize optimality. For the implementation of an optimal allocation it will not play a role. Equilibrium \( \langle \theta^*, s^*, w^*, u^* \rangle \) is given by the simultaneous solution to the job creation condition (3.14), the optimal search condition (3.20), either one of the two wage conditions (3.21) or (3.23), and the Beveridge curve (3.8). Observe that one can reduce the system further by eliminating the wage. Hence, equilibrium is given by the vector \( \langle \theta^*, s^*, u^* \rangle \) that solves

**(a) for sticky wages:**

JC-wage:

\[ \frac{y - w_0 a^\gamma}{r + \pi^x} = \frac{c \theta}{q^w(s,u,\theta)'}, \] (3.28)

JS-wage:

\[ \frac{w_0 a^\gamma - h - b/(1 - u) + k(s) - sk'(s)}{r + \pi^x} = \frac{sk'(s)}{q^w(s,u,\theta)'}, \] (3.29)

**(b) for flexible wages:**

JC-wage:

\[ \omega^f \left[ \frac{y - h - b/(1 - u) + k(s) - sk'(s)}{r + \pi^x} \right] = \frac{c \theta}{q^w(s,u,\theta)'}, \] (3.30)

JS-wage:

\[ \omega^w \left[ \frac{y - h - b/(1 - u) + k(s) - sk'(s)}{r + \pi^x} \right] = \frac{sk'(s)}{q^w(s,u,\theta)'}, \] (3.31)
and for both:

\[ BC : \frac{\pi^x}{\pi^x + q^w(s, u, \theta)} = u. \] (3.32)

If one combines both optimality conditions from the flexible wage regime one arrives at a convenient alternative condition for (3.31)

\[ sk'(s) = \frac{\omega}{1 - \omega} c \theta. \] (3.33)

This can alternatively be derived by combining (3.18), (3.22) and (3.11)

\[ sk'(s) = q^w(W - U) = \frac{\omega}{1 - \omega} q^w J = \frac{\omega}{1 - \omega} c \theta. \] (3.34)

The alternative condition (3.33) also allows us to write the wage equation in a slightly different way

\[ w = (1 - \omega) [z + T] + \omega (y + c \theta), \] (3.35)

which is well-known from Pissarides (2000).

### 3.4 Social optimum and implementation by benefits

This section derives the optimal allocation a social planner would choose.\(^{23}\) I will then discuss possible decentralizations of the allocation. As workers are risk-neutral, welfare and output maximization coincide\(^{24}\). The social planner faces the following recursive problem

\[
\Omega(N) = \max_{V, s} \left[ aF(N) + (L - N) [h - k(s)] - cV + \Omega(N') \right] \frac{1}{1 + r},
\] (3.36)

subject to (3.6). Note that the social planner in contrast to the agents does not take matching probabilities as given. Let \( S^S \equiv \Omega'(N) \) be the social value

\(^{23}\)In the canonical model, an equilibrium is unique if it exists. Sufficient conditions are constant returns to scale in the matching function and linear search technology, see Pissarides (2000). Only the second condition is fulfilled in the present model. Diamond (1982) and Diamond (1984) established that increasing returns in the matching function could generate multiple equilibria. Pissarides (1986) analyzes uniqueness and multiplicity of equilibria a more closely related model. It is important to realize that a social planner subject to the same matching technology faces the same indeterminacy concerning the allocation compared to the decentralized case. Obviously, he will pick the most efficient equilibrium. For the theoretical model I assume that agents can coordinate to do the same, i.e. I always focus on the most efficient equilibrium. Appendix 3.C establishes conditions for multiplicity and uniqueness. I also find that increasing returns are necessary. However, in the numerical part multiplicity of equilibria was not an issue.

\(^{24}\)Simply insert the resource constraint \( aF(N) - cV = N \pi + \Pi \), where \( \Pi \) denotes aggregate profits, to derive the utilitarian per-period welfare measure \( N \pi + (L - N) [h - k(s)] + \Pi \).
of filling an additional job, i.e. the social match surplus. Then the optimality conditions for \( V \) and \( s \) and the envelope condition for \( N \) evaluated in steady state are

\[
V : \quad -c + S^s \cdot \frac{\partial M}{\partial V} = 0 \tag{3.37}
\]

\[
s : \quad -k'(s)(L - N) + S^s \cdot \frac{\partial M}{\partial s} = 0 \tag{3.38}
\]

\[
N : \quad (1 + r)S^S = aF'(N) - [h - k(s)] + S^s \cdot \frac{\partial M}{\partial N} + (1 - \pi^x)S^S. \tag{3.39}
\]

I use the following derivations

\[
\frac{\partial M}{\partial V} = \zeta(1 - \eta)q^f, \quad \frac{\partial M}{\partial N} = -\zeta\eta q^w, \quad \frac{\partial M}{\partial s} = \zeta\eta \frac{M}{s}, \tag{3.40}
\]

which have to be inserted in (3.37) to (3.39). The first condition states that vacancies should be created up to the point where the share \( \eta_v = (1 - \eta)\zeta \) of the social value of the job equals the expected marginal costs of vacancy creation. The second condition implies that search is optimal if a share \( \eta_u = \eta \phi \zeta \) of the social value of a job equals the workers’ expected marginal search costs. Both can be written as

\[
\eta_v S^S = \frac{c\theta}{q^w}, \quad \eta_u S^S = \frac{s k'(s)}{q^w}. \tag{3.41}
\]

Combine both conditions to get the following simple relation

\[
s k'(s) = \frac{\eta}{1 - \eta} c\theta. \tag{3.42}
\]

Now combine the third and the second condition to get the social value of a job

\[
S^S = \frac{y - [h - k(s)] - sk'(s)}{r + \pi^x}. \tag{3.43}
\]

The social surplus is almost identical to the decentralized match surplus. The only difference is that UB can drive a wedge between them, i.e. \( S^S = S + \frac{b/(1 - u)}{r + \pi^x} \).

Consequently, the two social optimality conditions for vacancy creation and search intensity read

\[
\eta_v \left[ \frac{y - [h - k(s)] - s k'(s)}{r + \pi^x} \right] = \frac{c\theta}{q^w}, \tag{3.44}
\]

\[
\eta_u \left[ \frac{y - [h - k(s)] - s k'(s)}{r + \pi^x} \right] = \frac{s k'(s)}{q^w}. \tag{3.45}
\]
I will now show how these optimality requirements relate to the decentralized equilibrium conditions first in a sticky wage environment and then more extensively in the flexible wage regime.

### 3.4.1 Sticky wages and job rationing

The model of Landais et al. (2010) is nested in the specification above for $\xi = 1$, $\alpha < 1$ and using (3.21) as wage equation. They find that even if workers are risk-neutral there is a ‘job rationing’ externality implying too much search that should be corrected by a positive UB level. The idea of ‘job rationing’ can be easily explained within the framework of the presented model. I will focus on the job creation decision and restate the corresponding condition (3.28)

$$\frac{aF'(N) - w_{0}\gamma}{r + \pi x} = \frac{c}{q^{f}}.$$ 

It is useful to look at the limiting case $c \to 0$ such that posting vacancies is costless and search frictions vanish in the limit. In the canonical model with constant marginal productivity, i.e. $\alpha = 1$, firms would post vacancies ad infinitum such that $\theta \to \infty$ and matching from the workers’ perspective would occur instantaneously, i.e. $u \to 0$ and $N \to L$. In the model of Landais et al. (2010) this is not the case as jobs might be rationed at some point and no new jobs are created because the marginal product falls below the rigid wage.\(^{25}\) Clearly, rationing is more likely to occur if factor productivity $a$ is small, i.e. in a recession. It is easy to see that any additional uncoordinated search effort by the workers would be socially wasteful in such a situation.

In addition to decreasing returns to labor and wage stickiness there is another important deviation from the standard model in terms of the welfare measure. Landais et al. (2010) optimize workers’ welfare (or workers’ expected income in the case of risk-neutrality) but ignore aggregate profits for the derivation of the optimal UB formula in their stylized model.\(^{26}\) I will show that their results heavily depend on this last assumption. Hence, it should come at no surprise that they find that benefits should always be positive as the positive intergroup

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\(^{25}\)Actually, even with bargained, flexible wages jobs could be rationed as wages cannot fall below the workers per-period value of unemployment, while the marginal product can. Still, there is an important difference as this would be socially efficient rationing because a social planner would never want to maintain jobs that are less valuable than the value of home production.

\(^{26}\)For the numerical simulation of their complex, dynamic model they assume that profits can be taxed and redistributed. However, it is hard to isolate the exact role of the search correcting effect of the benefits as the simulations are just carry out for the case of risk-aversion.
externality of workers’ search effort on aggregate profits is ignored. In contrast, I assume that the social planner takes profits into account and maximizes total output. Observe that the allocation is efficient if and only if, by chance, the following conditions hold

\[ 1 - \eta = \frac{y - w_0 a^\gamma}{\Gamma^S} \quad \text{and} \quad b = 0, \quad \Gamma^S \equiv [y - (h - k(s)) - sk'(s)]. \tag{3.46} \]

\( \Gamma^S \) denotes the social per-period surplus, i.e. \( S^S = \frac{\Gamma^S}{r + \pi} \). Equation (3.46) is derived by comparing (3.28) and (3.29) with (3.44) and (3.45). It basically states the classical Hosios-condition, namely that the share of the surplus claimed by the firm is equal to the elasticity of the matching function w.r.t. \( v \). Clearly, the conditions stated in (3.46) differ from those derived by maximizing only the workers’ expected income and imply that for any value of \( a \) and \( \gamma \) in principle there could be too much or too little unemployment. Further, in case of failure of the first condition, UB cannot be used to restore efficiency.

The assumption of wage stickiness has been a controversially discussed topic in the macro-labor literature and while conventional wisdom on wage rigidity is mostly based on time-series analysis working with aggregate wage levels this view has been recently challenged by studies looking at individual worker data that allow for an explicit distinction between wages of new and continuing jobs. Most prominently Pissarides (2009) showed that the wage rigidity of continuing jobs is irrelevant for the job creation decision of firms as firms just care about their share of the expected surplus and not about how exactly the surplus is split in future periods. Indeed, individual-worker studies, like Haefke et al. (2008), estimate the elasticity of wages for continuing jobs in the range \( \epsilon_{wa}^c \in [0.3, 0.5] \) but when they estimate the same elasticity for newly created jobs they find that those move much stronger with the cycle. Combining their point estimate with a more indirect estimate by Pissarides (2009) himself gives a range of \( \epsilon_{wa}^n \in [0.9, 1.02] \). Recall that applying the typical bargaining sharing rule results in \( \epsilon_{wa}^w \approx 0.98 \) which is perfectly in line with the corresponding estimates. As wage stickiness during an employment spell does not affect the job creation condition, applying the bargaining sharing rule for all jobs should be preferred over using sticky wage specifications. This basically summarizes the argument of Pissarides (2009) which I will refer to as the ‘Pissarides-critique’ concerning wage stickiness. Gertler and Trigari (2009) and Blanchard and Gál (2010) responded to the ‘Pissarides-critique’ by arguing that the true elasticity of wages for new jobs w.r.t. productivity is hard to identify as the high estimates
could in principle also stem from compositional effects as Haefke et al. (2008) did not control for occupational changes\textsuperscript{27}. Nevertheless, to my knowledge no empirical study has convincingly confirmed real wage rigidity for newly created jobs which is why I will focus on the flexible wage specification as recommended by Pissarides (2009).

### 3.4.2 Flexible wages and non-constant returns to matching

Let me return to the optimality conditions (3.44) and (3.45). Compare this expressions to the decentralized conditions for flexible wages (3.30) and (3.31). Both condition pairs differ w.r.t. two attributes: the size of the surplus and shares of the surplus the firm and the worker receive. The social and the decentralized surplus just differ by a ‘wedge’ created if $b \neq 0$, i.e. $S^S = S + \frac{b/(1-u)}{r+\pi^x}$. The socially optimal shares are influenced by the degree of returns to scale $\xi$, while the decentralized shares depend on the surplus tax $t$. Take $t = b = 0$ for the moment. It is obvious that in case of constant returns, i.e. $\xi = 1$, the classical Hosios (1990)-condition $\omega = \eta = \eta_u$ implies efficiency. Now let us assume $\xi \neq 1$. The social planner takes congestion effects on the probability of finding a job through changes in the level of unemployment into account, which is not done by agents in the decentralized economy who naively treat $q^w$ and $q^f$ as given. As the classical Hosios-condition for constant returns to scale is given by $\omega = \eta$, this parameter constellation will serve as my benchmark in order to isolate the additional inefficiencies generated by $\xi \neq 1$.

**Proposition 3.4.1. Generalized Hosios-condition:** In a regime with flexible wages the allocation is efficient if the effective bargaining powers and the total matching elasticities coincide, i.e. $\omega^w = \eta_u$ and $\omega^f = \eta_v$, and no other policy is in place. Given that the classical Hosios-condition, i.e. $\omega = \eta$, is fulfilled, the efficient allocation can be reached by setting a surplus tax/subsidy $t = 1 - \xi$ and $b = 0$.

**Proof.** This follows directly from comparing (3.30) and (3.31) with (3.44) and (3.45). Note that a failure of $\omega \neq \eta$ cannot be corrected by adjusting $t$ as efficiency along the job creation margin would require $t = 1 - \xi \frac{1-\eta}{1-\omega}$ while optimality along the job search margin demands $t = 1 - \xi \frac{\eta}{\omega}$\textsuperscript{28}.

Observe that the generalized Hosios-condition collapses to its classical form if $\xi = 1$. Given that $\omega = \eta$ and in absence of any policy instrument one can

\textsuperscript{27}The argument is that in bad times an architect might be forced to take a low paying job as a cab driver, while if he found a job in an architectural firm he would have suffered a much smaller than almost proportional loss due to the fall in aggregate productivity.

\textsuperscript{28}See for example chapter 1 of this thesis for a discussion on how to correct in the case of $\omega \neq \eta$. 
distinguish three cases. If $\xi = 1$ (constant returns) the level of employment is efficient. If $\xi < 1$ (decreasing returns) the level of employment is inefficiently high. If $\xi > 1$ (increasing returns) the level of employment is inefficiently low. This implies that with decreasing returns one requires an implicit employment tax to prevent excessive aggregate search, while in case of increasing returns one requires an implicit employment subsidy to foster aggregate search. How should the generalized Hosios-condition be interpreted? Recall that the classical Hosios-condition finds the optimal trade-off between the positive intergroup and the negative intragroup externalities. This is done by splitting the match surplus in an optimal way. If $\omega > \eta$ then workers would search too much while firms would exert too little search effort. The generalized version of the Hosios-condition in addition gives a formula for optimal aggregate search. If $\xi < 1$ then both, workers and firms, would provide too much search effort from a social perspective. That is why the surplus itself has to shrink in order to reduce search incentives for all agents. Importantly, this can only be achieved through policy intervention while a constant returns to scale setting in principle allows equilibrium to be efficient without any intervention. Proposition 3.4.1 presents results for an employment tax/subsidy in form of a proportional surplus tax/subsidy as originally derived by Keller et al. (2010). The idea of a direct surplus tax is admittedly very abstract. This becomes even more clear with the following proposition.

**Proposition 3.4.2. Impossibility of proportional surplus taxation:** There is no combination of proportional taxes on any economic activity flows that could mimic surplus taxation of the form (3.26).

The proof is provided in the appendix. I imposed the reasonable assumption that a government cannot directly tax the match surplus. Even if it could tax any economic activity flow like output, wages, benefits, vacancy posting costs, search costs or home production, proposition 3.4.2 states that proportional surplus taxation cannot be implemented. The intuition is that one would require proportional subsidization of vacancy posting or marginal search costs on one hand to tax the surplus proportionally while on the other hand one must not allow for such instruments to prevent distortion of the marginal cost components on the right hand side of (3.26).
I have argued that the implementation of an efficient allocation using a proportional surplus tax/subsidy is impossible as a typical policy maker usually does not have an instrument like that at his disposal and it cannot be mimicked by other instruments. Instead of changing the *splitting weights* of the surplus to achieve proportional surplus taxation/subsidization one could directly change the *size* of the surplus by controlling the wedge between the social and the decentralized surplus. In this framework this can be done by setting UB accordingly. I still assume $\omega = \eta$ to isolate the additional from the conventional search externalities. Set $t = 0$ and subtract (3.44) from (3.30) and solve for the optimal UB level. Equation (3.47) gives the optimal size of UB in absence of any insurance motive.\(^{32}\)

\[
b = \Gamma^S (1 - \xi)(1 - u). \tag{3.47}
\]

Observe that this is indeed the optimal level of $b$ as it also implies optimal search intensity. If $\xi = 1$ the optimal level of benefits is $b = 0$. If $\xi > 1$ it is $b < 0^{33}$ which implies that otherwise there would be too little search. In case of decreasing returns, $\xi < 1$, there is excessive search which requires $b > 0$. Another important implication of (3.47) is that for $\xi \neq 1$ the optimal $b$ has to move proportionally with the surplus itself. The results are summarized in the following proposition.

**Proposition 3.4.3. Optimal unemployment benefits:** The allocation is efficient if unemployment benefits are set according to (3.47). In case of decreasing (increasing) returns to scale optimal UB are positive (negative) and rise (fall) with productivity.

The proof is provided in the appendix and simply relies on the trivial property that both the employment rate $1 - u$ and the social per-period surplus $\Gamma^S$ are pro-cyclical. The different cases are illustrated in figure 3.1.

To summarize, if $\xi < 1$ then $b > 0$ and $\frac{db}{da} > 0$ i.e. benefits have to be set pro-cyclically. On the other hand if $\xi > 1$ then $b < 0$ and $\frac{db}{da} < 0$, i.e. benefits should be counter-cyclical. The finding of Landais et al. (2010) that benefits should be positive and counter-cyclical are a mixture the results from both

\(^{32}\)In the present framework UB are adjusted directly. Appendix section 3.D derives the optimal formula for the replacement rate.

\(^{33}\)Here I assume that the policy maker can set $b < 0$ i.e. he can tax home production, which is typically not a feasible option. However, recall that the set-up with risk-neutral workers was done to simplify the analysis and isolate the Pigouvian role of UB. In a richer model with risk-aversion and positive unemployment insurance increasing returns to scale would rather imply a reduction in existing UB instead of negative UB, which is obviously feasible.
3.5 A numerical example and risk-aversion

This section serves to get an impression of how big the search correction actually has to be and how this quantitatively relates to the typical insurance provision function. I will therefore extend the existing model to allow for non-linear utility. Risk-aversion is introduced by wrapping per-period consumption flows into a felicity function $u(\cdot)$ with the properties $u'(\cdot) > 0$ and $u''(\cdot) < 0$.\footnote{Note that because of convention I use the notation $u(\cdot)$ for the instantaneous utility function despite the fact that $u$ also denotes the unemployment rate. The distinction becomes clear from the context.} I assume that utility from consumption and disutility from search effort\footnote{Note that $k(\cdot)$ therefore might have a different functional form than before.} are linearly separable. One can write the recursive values of being unemployed and employed as

$$U_i = \max_{s_i} \left[ u(h + b) - k(s_i) + q_i^w W_i + (1 - q_i^w) U_i \right] \frac{1}{1+r}, \quad (3.48)$$

$$W_i = \left[ u(w_i - T) + \pi^x U_i + (1 - \pi^x) W_i \right] \frac{1}{1+r}. \quad (3.49)$$

returns to scale scenarios. With decreasing returns to scale there is too much aggregate search, hence UB should be positive like in Landais et al. (2010), but in contrast benefits should rise in good times as unemployment shrinks which reduces the effectiveness of the matching process. If returns are increasing, the opposite is true and UB should be counter-cyclical because matching is less effective if the level of unemployment is high, as in Landais et al. (2010), while too little aggregate search implies a negative optimal level of UB.

Figure 3.1: Optimal unemployment benefits over the business cycle for different degrees of returns to scale
It is easy to see that the optimal job search condition is given by
\[
\frac{u(w - T) - u(b + h) + k(s) - sk'(s)}{r + \pi x} = \frac{sk'(s)}{q^w}.
\] (3.50)

For the determination of the wage I will again resort to a simple surplus sharing rule.
\[
\tilde{W} - \tilde{U} = \omega_1 - \omega J.
\] (3.51)

Here \(\tilde{W}\) and \(\tilde{U}\) denote the values of working and not working in pure monetary terms. I assume that utility is non-transferable, i.e. the two parties can only share the surplus evaluated in monetary terms. See Michau (2011) for more details on this surplus splitting rule.\(^{36}\) Consequently, the wage equation does not change in comparison to the risk-neutral case and is again given by (3.23). Therefore also the condition for optimal vacancy posting is unchanged. Equilibrium is given by the vector \(\langle \theta^*, s^*, w^*, u^*, T^* \rangle\) that solves the following system of equations

\[\begin{align*}
JC: & \quad \frac{y - w}{r + \pi x} = \frac{c\theta}{q^w}, \\
JS: & \quad \frac{u(w - T) - u(b + h) + k(s) - sk'(s)}{r + \pi x} = \frac{sk'(s)}{q^w}, \\
\text{wage:} & \quad (1 - \omega) \left[ z + T - sk'(s) \right] + \omega y = w, \\
\text{government:} & \quad \frac{u}{1 - u} b = T, \\
BC: & \quad \frac{\pi x}{\pi x + q^w} = u.
\end{align*}\] (3.52-3.56)

I now characterize the optimal allocation. To simplify the analysis I use \(\alpha = 1\) such that the representative firm does not make profits. As an implementation of the first best allocation requires full insurance which is incompatible with bargained wages with positive bargaining power for the workers I will consequently look for a second best solution. In this case the planner maximizes utilitarian welfare subject not only to the law of employment but also to the implementability constraints given by the decentralized equilibrium conditions. For reasons of analytic convenience I will write the corresponding Bellman equation as a function of the unemployment rate instead of total employment.

\(^{36}\)A similar result can be derived when using a first-order Taylor approximation of the first-order condition of an explicit Nash bargaining game as done in chapter 2.
The second best optimization problem in recursive form looks as follows

\[ \Theta(u) = \max_{\theta,s,b,w,T} \left[ L(1 - u) \cdot u(w - T) + Lu \cdot [u(h + b) - k(s)] + \Theta(u') \right] \frac{1}{1 + r'}, \]

subject to \( u' = (1 - q^w)u + \pi x(1 - u), \) (3.57)

and the system of equilibrium conditions (3.52) to (3.55). As the economic interpretability of the resulting first-order conditions is rather limited I will illustrate the case of risk-aversion using a small numerical example.

### 3.5.1 Calibration

First, the following functional forms for the utility function and the search effort function where chosen

\[ m(su,v) = (su)^\eta v^{1-\eta}, \] (3.58)
\[ \Phi(x) = M_0 x^\xi, \] (3.59)
\[ k(x) = k_0 x^\nu, \quad \nu > 1, \] (3.60)
\[ u(x) = \begin{cases} x^{1-\sigma-1} + 1 & \text{if } \sigma \geq 0 \text{ and } \sigma \neq 1, \\ \ln(x) + 1 & \text{if } \sigma = 1. \end{cases} \] (3.61)

Observe that \( \sigma = 0 \) implies \( u(x) = x \) as used in the previous sections. The calibration mainly relies on the values chosen in Pissarides (2009). The benchmark calibration is done for a ‘typical’ separated labor market in the United States at quarterly frequency. Table 3.2 summarizes the choices for the parameters for the benchmark case. Worker flows, labor market tightness and consequently unemployment are taken from the Job Openings and Labor Turnover Survey (JOLTS) and the Help-Wanted Index (HWI) as reported in Pissarides (2009) and Shimer (2005a). Hence, the average separation rate \( \pi x = 0.036 \), job finding rate \( q^w = 0.594 \) and labor market tightness \( \theta = 0.72 \) for the periods 1960 to 2004 were targeted. As the evidence for the returns to scale in matching is not persuasively conclusive, I will base the benchmark calibration on the assumption \( \xi = 1 \). Deviations from the constant returns to scale scenario will then be introduced by setting \( \xi \) to the boundary values \( \xi \in \{0.5, 1.5\} \). To target the same unemployment rate I will recalibrate the efficiency parameter of the matching function \( M_0 \) accordingly. Chetty (2006b) estimates the parameter of relative risk-aversion close to 1. Chetty and Szeidl (2007) argue that for small shocks such as an unemployment spell this value could be considerably bigger. I set the coefficient of risk-aversion to \( \sigma = 2 \). Nickell et al. (2005) report that a 10\% increase in the replacement ratio leads to an increase of unemployment by
1.11%-points. I chose the value for the elasticity of the search cost function $\nu$ to be 3. This slightly overestimates the responsiveness reported by Nickell et al. (2005) but implies that the corrected unconditional matching elasticity is not unreasonably large. Appendix section 3.B explains this correction procedure. The correction has to be made because search intensity is usually unobserved which implies that estimates of the matching function elasticities take the endogenous adjustment of search intensity into account which is not done by the deep parameter $\eta$. Taking an estimate of $\tilde{\eta} = 0.4$ for the United States which comes close to the values reported by Anderson and Burgess (2000) implies that $\eta = \tilde{\eta} \frac{\nu}{\nu - 1} = 0.6$. The bargaining power is then set accordingly to $\omega = 0.6$ to rule out any additional unbalanced search externalities that would add on top to the ones created in the presented non-constant returns to scale scenarios. In the benchmark calibration I rule out decreasing marginal productivity and set the marginal product to $y = a = 1$. In equilibrium this will imply a wage that is only slightly lower than 1. A critical choice is the one of the value of unemployment $z = b + h - k(s)$. Although I will look at optimal UB I need a value for currently implement benefits that lead to the observed unemployment rate. The OECD tax benefit calculator gives an initial replacement rate of approximately 50% to 70% depending on personal characteristics like marital status and number of children, etc. Benefits last for 6 months. Afterwards the unemployed worker receives other social assistance payments. To take this loss of unemployment income into account I target the lower bound of the tax benefit calculator’s outcome and set $b = 0.5$. Hall and Milgrom (2008) derive a total value of unemployment of $z = 0.71$ that I will target as well, i.e. $h - s(k) = 0.21$. It is not clear how much weight to put on pure home production $h$ on one side and the effort costs of search $k(s)$ on the other side. Hagedorn and Manovskii (2008) argue that home production might be substantial at the same time $b + h$ must not exceed the wage $w$ because I want to rule out an equilibrium where no search, i.e. $s = 0 \Rightarrow k(0) = 0$, is optimal. I set $h = 0.3$ and $k_0 = 0.05$ to comply with these constraints.

Table 3.4 in the appendix summarizes all calibrations for $b = 0.5$ for different combinations of coefficients of constant relative risk-aversion, returns to scale in the matching function and degrees of decreasing marginal productivity in order to match $q^w$ and consequently $u$ of our benchmark by adjusting $M_0$.

$^{37}$Note that the correction formula was derived from the optimal job search condition of risk-neutral workers while the benchmark calibration is done for the case of risk-aversion. The correction is therefore not completely precise but should give a reasonable value for the unconditional elasticity.
3.5. A NUMERICAL EXAMPLE AND RISK-AVERSION

Table 3.2: Benchmark calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>0.004</td>
<td>Pissarides (2009)</td>
</tr>
<tr>
<td>(\pi^x)</td>
<td>0.036</td>
<td>Shimer (2005a)</td>
</tr>
<tr>
<td>(\nu)</td>
<td>3</td>
<td>(\epsilon^u \approx 1.1), Nickell et al. (2005)</td>
</tr>
<tr>
<td>(\eta)</td>
<td>0.6</td>
<td>(\bar{\eta} = 0.4), Anderson and Burgess (2000)</td>
</tr>
<tr>
<td>(\omega)</td>
<td>0.6</td>
<td>no additional inefficiency</td>
</tr>
<tr>
<td>c</td>
<td>0.22</td>
<td>average (\theta), Pissarides (2009)</td>
</tr>
<tr>
<td>b</td>
<td>0.5</td>
<td>OECD tax benefit calculator</td>
</tr>
<tr>
<td>h</td>
<td>0.3</td>
<td>(b + h) almost at (w), Hagedorn and Manovskii (2008)</td>
</tr>
<tr>
<td>(M_0)</td>
<td>0.5973</td>
<td>job finding probability, Shimer (2005a)</td>
</tr>
<tr>
<td>(k_0)</td>
<td>0.05</td>
<td>(z = 0.71), Hall and Milgrom (2008)</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>2</td>
<td>Chetty (2006b) and Chetty and Szeidl (2007)</td>
</tr>
<tr>
<td>(\xi)</td>
<td>1</td>
<td>returns to scale, benchmark</td>
</tr>
<tr>
<td>(y = a)</td>
<td>1</td>
<td>normalization</td>
</tr>
</tbody>
</table>

Accordingly.\(^{38}\)

3.5.2 Numerical results

To study the effects over the business cycle I used shocks to factor productivity \(a\) of plus and minus 10%, which are clearly at the upper bound of reasonable values. Table 3.3 states optimal UB for different combinations of productivity shocks and assumptions concerning returns to matching and relative risk-aversion.

Table 3.3: Optimal unemployment benefits over the cycle

<table>
<thead>
<tr>
<th>(\sigma)</th>
<th>(\xi)</th>
<th>(a)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9</td>
<td>1.0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0407</td>
<td>0.0437</td>
</tr>
<tr>
<td>0</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.0136</td>
<td>-0.0145</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3338</td>
<td>0.3921</td>
</tr>
<tr>
<td>2</td>
<td>0.3213</td>
<td>0.3785</td>
</tr>
<tr>
<td>1.5</td>
<td>0.3171</td>
<td>0.3740</td>
</tr>
</tbody>
</table>

Three important findings can be inferred from those results. First, the level of optimal Pigouvian UB to correct for excessive or too little aggregate search is rather small. For the boundary values of reasonable choices of \(\xi\) I find that for decreasing returns to scale benefits have to be set to about 4.4% of the wage to correct for excessive search while for increasing returns UB have be to -1.5% of the wage. Second, the responsiveness of optimal Pigouvian UB over the cycle

\(^{38}\)This procedure obviously eliminates all effects of different choices of market size \(L\) which are absorbed by \(M_0\). For the presented values of \(M_0\) a local labor market size of \(L = 1000\) was assumed.
is very mild given the size of the productivity shocks. From the worst to the best productivity state UB vary from 0.0407 to 0.0465 if returns are decreasing and from -0.0136 to -0.0153 if returns are increasing. And third, in case of risk-aversion one can establish a strong pro-cyclicality of UB due to the within-period insurance motive which overturns the counter-cyclicality due to search correction in the case of increasing returns to matching. I will comment on all three findings in the rest of this section.

The simulation results show that the variation of optimal \( b \) over the cycle is quantitatively not very important. But this result might be a direct consequence of the Shimer (2005a)-puzzle. Recall that optimal \( b \) has to move with \( \Gamma_S \), the social per-period surplus, as restated in (3.62) for the case of risk-neutral workers.

\[
b = (1 - \bar{\xi})\Gamma_S(1 - u).
\]

(3.62)

Shimer argues that the canonical model cannot reproduce the comparably large elasticity of labor market tightness \( \theta \) w.r.t. labor productivity shocks \( a \) that is observed in the data. While the observed elasticity is \( \epsilon^\theta_a = 7.56 \) the model only predicts \( \epsilon^\theta_a = 1.71 \) for an otherwise reasonable calibration. Expressing (3.62) in terms of elasticities\(^{39}\) gives

\[
\epsilon^b_a = \left[ (1 - \bar{\eta}_v) + \frac{\bar{\eta}_v u}{1 + (\bar{\xi} - 1)(1 - u)} \right] \cdot \epsilon^\theta_a. \tag{3.63}
\]

Clearly, if \( \epsilon^\theta_a \) would be blown up by the factor 4.42 to match the data also optimal benefits should react stronger to the business cycle by the same factor. Obviously, if one thinks that the Shimer-Puzzle stems from a misspecification of the model it has to be pointed out that the same model was used to derive the optimal benefit formula. However, if the low sensitivity merely results from an unrealistic calibration, as argued by Hagedorn and Manovskii (2008), then (3.63) is correctly specified and the sensitivity of optimal UB to the cycle can be amplified by e.g. increasing the value of home production. This would have two consequences in the model. On one hand, wages become less flexible which boosts the variability of the match surplus and hence would imply more variation in optimal UB over the cycle. On the other hand, the surplus as such becomes smaller and so does the level of optimal UB. Hence, changing the calibration as suggested by Hagedorn and Manovskii (2008) would trade-off my findings one and two.

\(^{39}\)The derivation is provided in appendix 3.A.
Finding three stems from the strong pro-cyclicality requirement for the benefits to insure the workers against unemployment risk within a period. The reason is that wages strongly co-move with the cycle which would in good times increase the difference of income from employment versus income from unemployment if latter was constant. Due to their sticky wage assumption this effect is less pronounced in Landais et al. (2010). Observe that in the current set-up intertemporal insurance and consumption smoothing between periods is ignored as I just compare steady states with different levels of aggregate productivity and assume that government budget has to be balanced period by period. Introducing a motive for intertemporal insurance would require a full dynamic, stochastic framework beyond the scope of this paper which tries to focus on the Pigouvian character of UB rather then the provision of insurance. Assuming that the government - in contrast to the workers - faces an intertemporal budget constraint that allows to shift resources over time it would mimic precautionary savings of the workers to smooth their consumption over the cycle. Hence, intertemporal insurance would demand counter-cyclical benefits. This might counteract the strong pro-cyclicality found for within period insurance.

So far I assumed that marginal productivity is independent of the level of employment. Relaxing this assumption as done in Landais et al. (2010) introduces a stronger sensitivity of the model to changes in the parameterization that requires some recalibration effort. However, it does not imply an amplification of the surplus sensitivity as demonstrated by the results in table 3.5. The intuition is straightforward. A positive shock to productivity directly increases the surplus and therefore employment. But the increase in employment reduces the marginal product and therefore dampens the total effect. Hence, in contrast to Landais et al. (2010) the degree of decreasing marginal productivity interacts with the extent of the search externality in a negative way.

3.6 Conclusion

Beside the function of insuring workers against the risk of unemployment, unemployment benefits might also be used as a Pigouvian instrument. When the magnitude of distorted aggregate search depends on the state of the economy along the business cycle, unemployment benefits optimally have to vary over the cycle too, even if workers are risk-neutral. The paper presents a Diamond-Mortensen-Pissarides model that explicitly allows for non-constant returns to matching that generate this type of externalities as the additional level effects of
unemployment on the match efficiency are not taken into account by the agents. The presented model nests the model of Landais et al. (2010) who find that unemployment benefits should always be positive and even more so in bad times even if the insurance provision motive is neglected. It is shown that their result is sensitive to the choice of the welfare criterion. After taking aggregate profits into account and dropping the criticized assumption of wage rigidity for new hires, I derive a generalized Hosios-condition guaranteeing constrained efficiency. If returns to scale are non-constant another dimension of search externalities related to the aggregate amount of search effort is introduced that requires government intervention in any case. The implementation of the optimal allocation involves pro-cyclical (counter-cyclical) benefits correcting for excessive (too little) aggregate search activity if the matching function exhibits decreasing (increasing) returns to scale. In a numerical exercise I introduce risk-aversion to assess the role of Pigouvian benefits compared to the typical function of insurance provision. It is shown that the quantitative role is rather limited. However, this conclusion might be premature in light of the fact that the low variability of the optimal unemployment benefits is directly connected to the implausibly low responsiveness of the match surplus to productivity shocks for this class of models, an enigma known as the Shimer-puzzle.
Appendix

3.A Derivations and proofs

Derivations of the elasticities in section 3.2.1. Note that
\[ \frac{\partial \Phi(Lu \cdot m(s, \theta))}{\partial Lu} = \frac{\Phi(Lu \cdot m(s, \theta))}{Lu \cdot m(s, \theta)} \cdot \frac{\partial m(s, \theta)}{\partial \theta} \cdot (1 - \eta). \]
\[ \frac{\partial k}{\partial \theta} = \frac{m(s, \theta)}{\partial \theta} \cdot \xi + \frac{\partial k}{\partial \theta} = \frac{1}{\xi} \frac{\partial m}{\partial \theta} \cdot \xi + \frac{\partial k}{\partial \theta} = \frac{1}{\xi} \frac{\partial m}{\partial \theta} \cdot \xi \cdot \eta. \]
\[ \frac{\partial q^w(s, u, \theta)}{\partial \theta} = \frac{1}{Lu} \cdot \frac{\partial \Phi(Lu \cdot m(s, \theta))}{\partial Lu} \cdot Lu \cdot \frac{\partial m(s, \theta)}{\partial \theta} \]
\[ = \frac{\Phi(Lu \cdot m(s, \theta))}{Lu \cdot m(s, \theta)} \cdot \frac{\partial m(s, \theta)}{\partial \theta}(1 - \eta) \]
\[ = (1 - \eta)\xi \frac{q^w}{\partial \theta} = (1 - \eta)\xi q^f = \eta q^f. \]
\[ \text{Note that } \Phi_u \equiv \frac{\partial \Phi(Lu \cdot m(s, \theta))}{\partial u} = \xi q^w L \text{ and } \Phi_L \equiv \frac{\partial \Phi(Lu \cdot m(s, \theta))}{\partial L} = \xi q^w u. \]
\[ \frac{\partial q^w(s, u, \theta)}{\partial u} = \frac{\Phi' \cdot Lu - L \cdot M}{L^2u^2} = \frac{\xi q^w L \cdot Lu - L \cdot L}{L^2u^2} \]
\[ = (\xi - 1)q^w u^{-1} \]
\[ \frac{\partial q^w(s, u, \theta)}{\partial L} = (\xi - 1)q^w L^{-1}. \]

Proof of proposition 3.4.3. The optimal UB formula given by (3.47) is derived as follows. First, compare (3.26) and (3.41). Given that \( \omega = \eta \) and \( t = 0 \) efficiency is restored if and only if \( \xi S^S = S \) or \( \xi \Gamma^S = \Gamma \). Use the relation of both per-period surpluses \( \Gamma = \Gamma^S - b/(1 - u) \) and solve for \( b \). To prove the last statement of proposition 3.4.3 one simply has to show that \( \Gamma^S \) and \( 1 - u \) are pro-cyclical. First, note the following relation of the social and the decentralized per-period surplus
\[ \Gamma^S = S^S(r + \pi^x) = S(r + \pi^x) + \frac{b}{1 - u} \equiv \Gamma + \frac{b}{1 - u}. \] (3.64)
Insert the optimal benefits (3.47) in the job creation condition \( (1 - \omega) \frac{\Gamma}{r + \pi^x} = \frac{c}{q^f} \) to get
\[ (1 - \omega)\xi \frac{\Gamma^S}{r + \pi^x} = \frac{c}{q^f}. \] (3.65)
Next, eliminate search intensity \( s \) by inserting for \( k(s) - sk'(s) = (\nu - 1)k(s) = \)
\[ \frac{v-1}{v} \frac{\omega}{1-\omega} c \theta \] and use the unconditional probability\(^{40}\) \(\tilde{q}_f\). Rearrange to get
\[ y - h = \frac{r + \pi^x}{(1-\omega)\xi} c + \frac{v-1}{v} \frac{\omega}{1-\omega} c \theta. \tag{3.66} \]

Clearly, when \(a\) increases \(y\) and consequently the left-hand side have to rise. As the right-hand side is increasing in \(\theta\)\(^{41}\), I have established that \(d\theta / da > 0\). Applying this relation to (3.65) and the Beveridge curve using unconditional job finding probabilities implies that \(d\Gamma^S / da > 0\) and \(d(1-u) / da > 0\) which completes the proof.

Derivations of the elasticity in section 3.5. First take the total differential of (3.47).
\[ db = (1-\xi)(1-u) d\Gamma^S + (1-\xi)\Gamma^S d(1-u) \quad \Leftrightarrow \quad \epsilon^b_a = \epsilon^{\Gamma^S}_a + \epsilon^{1-u}_a. \tag{3.67} \]

I will rewrite (3.67) in terms of \(\epsilon^\theta_a\) but before that I will contrast the social per-period surplus \(\Gamma^S\) with the decentralized per-period surplus \(\Gamma\)
\[ \Gamma \equiv \left[ y - (h - b/(1-u) - k(s)) - sk'(s) \right] \quad \text{such that} \quad \Gamma^S - \frac{b}{1-u} = \Gamma. \tag{3.68} \]

Inserting the optimal benefits gives \(\xi \Gamma^S = \Gamma\), i.e. at the optimum benefits will not only force the decentralized per-period surplus to coincide with the optimal share of the social per-period surplus but also both surpluses will move over the cycle in a synchronized way, i.e. \(\epsilon^{\Gamma^S}_a = \epsilon^\Gamma_a\). Rewrite the job creation condition, take the total differential and rearrange to get
\[ \epsilon^\Gamma_a = (1-\tilde{\eta}_w)\epsilon^\theta_a = \epsilon^{\Gamma^S}_a, \tag{3.69} \]

where I used the unconditional vacancy filling probability \(\tilde{q}_f\)!\(^{42}\). Next, one has to transform \(\epsilon^{1-u}_a\) which is equal to \(\epsilon^\theta_a \cdot \epsilon^{1-u}_a\). The derivative of \(1-u = \frac{\tilde{q}^w(u,\theta)}{\pi^x + \tilde{q}^w(u,\theta)}\) w.r.t. \(a\) can be written as
\[ \frac{d(1-u)}{da} = \frac{\partial(1-u)}{\partial \tilde{q}^w} \cdot \frac{\partial \tilde{q}^w}{\partial \theta} \cdot \frac{d\theta}{da} - \frac{\partial(1-u)}{\partial \tilde{q}^w} \cdot \frac{\partial \tilde{q}^w}{\partial u} \cdot \frac{d(1-u)}{da}, \tag{3.70} \]

where I used the fact that \(\frac{du}{da} = -\frac{d(1-u)}{da}\). The first term in (3.70) can be com-
puted as follows
\[
\frac{\partial (1 - u)}{\partial \theta} = \frac{\eta v \theta^{-1} \bar{q}^w \pi^x + \bar{q}^w}{\pi^x + \bar{q}^w} - \frac{\eta v \theta^{-1} (\bar{q}^w)^2}{\pi^x + \bar{q}^w} = (1 - u) \eta v \theta^{-1} [u + 1 - u] - \eta v \theta^{-1} (1 - u)^2
\]
(3.71)
\[
= \eta v \theta^{-1} (1 - u) u. \]

Proceed analogously to get \( \frac{\partial (1 - u)}{\partial u} = (\xi - 1)(1 - u) \). Both can now be combined to get
\[
\epsilon_a^{1-u} = \frac{\eta v u}{1 + (\xi - 1)(1 - u)} \cdot \epsilon_a^\theta. \quad (3.72)
\]

In total the elasticity of optimal \( b \) w.r.t. \( a \) is therefore given by
\[
\epsilon_a^b = \left[(1 - \eta v) + \frac{\eta v u}{1 + (\xi - 1)(1 - u)}\right] \cdot \epsilon_a^\theta. \quad (3.73)
\]

**Proof of proposition 3.4.2.** I introduce proportional taxes/subsidies for all flow variables: output, value of leisure, vacancy posting costs, the worker’s and the employer’s wage rate. Denote the corresponding proportional tax rates \( t^y, t^z, t^c, t^w \) and \( t^f \). First, assume that our surplus sharing condition (3.22) is unaffected. Rearrange to get
\[
\omega (1 - \omega) r J - (1 - \omega) r U = - (1 - \omega) r U \text{ and insert to get}
\]
\[
\omega = \frac{\omega (1 - t^y) y + (1 - \omega) (1 - t^z) z + \omega (1 - t^c) c \theta}{\omega (1 - t^f) + (1 - \omega) (1 - t^w)}. \quad (3.74)
\]

Inserting in the job creation condition with taxes,
\[
\frac{(1 - t^y) y - (1 - t^f) w}{r + \pi^x} = (1 - t^c) \frac{c \theta}{\bar{q}^w}, \quad (3.75)
\]
and analyzing the condition for optimal search with taxes,
\[
(1 - t^c) c \theta \frac{\omega}{1 - \omega} = (1 - t^z) sk'(s), \quad (3.76)
\]
reveals that for no values of our taxes one can mimic (3.26). Now consider the case where the sharing rule is affected by taxation because I explicitly derive it from Nash bargaining. This implies
\[
\omega (1 - t^w) r J - (1 - \omega) (1 - t^f) r W = - (1 - \omega) (1 - t^f) r U. \quad (3.77)
\]
The wage would then be given by
\[ w = \frac{\omega}{1 - t_f} \left[ (1 - t^y)y + (1 - t^c)c\theta \right] + \frac{1 - \omega}{1 - tw} (1 - t^z)z. \] (3.78)

Again, after inserting in (3.75) it is easy to verify that there is no way to implement (3.26).

3.B Conditional versus unconditional matching function

The model treats search intensity as an endogenous variable which enters the matching function and itself is a function of \( u \) and \( v \). As search intensity is very hard to measure, empirical studies typically only estimate elasticities w.r.t. \( u \) and \( v \) neglecting search intensity. Those estimates then take endogenous responses of \( s \) implicitly into account and therefore do not coincide with their theoretical counterparts \( \eta_u \) and \( \eta_v \) that measure changes in the number of matches for constant search intensity. Putting a little bit of structure on the search cost function will allow to correct the estimated elasticities and relate them to the deep parameters of the model. As an illustration, this is only done for the case of risk-neutrality as the optimal job search condition simplifies considerably in that case. Assume that the function \( k(\cdot) \) is characterized by a constant elasticity \( \epsilon^k_s = \nu \). Consequently, also the function \( \kappa(s) \equiv sk'(s) \) will have a constant elasticity \( \epsilon^\kappa_s = \nu \). Rewrite the optimal job search condition (3.33) as
\[ s = \kappa^{-1} \left( \frac{\omega}{1 - \omega} c\theta \right) \equiv s(\theta). \] (3.79)

This implies the following elasticities \( \epsilon^{s(\theta)}_s = 1/\nu \) and \( \epsilon^{k(s(\theta))}_s = 1 \). Now eliminate \( s \) by inserting (3.79) in the function for aggregate search activity \( Lm(su, v) \) conditional on \( s \). This results in an unconditional function \( \tilde{L}m(u, v) \) with the following elasticities
\[ \epsilon^{\tilde{m}}_u = \eta - \eta \frac{\nu}{v} \equiv \tilde{\eta}_u \quad \text{and} \quad \epsilon^{\tilde{m}}_v = 1 - \eta + \eta \frac{\nu}{v} \equiv 1 - \tilde{\eta}. \] (3.80)

Clearly, also the unconditional aggregate search activity function has constant returns to scale. The elimination of \( s \) just implies a re-weighting of the elasticities in favor of the vacancy rate. Further, the total matching elasticities are simply \( \epsilon^{M}_u = \tilde{\eta} \cdot \tilde{\xi} \equiv \tilde{\eta}_u \) and \( \epsilon^{M}_v = (1 - \tilde{\eta}) \cdot \tilde{\xi} \equiv \tilde{\eta}_v \). Thus the degree of returns to scale is independent of whether one uses the conditional or the unconditional matching function. This is obviously a direct consequence of the invariance of the labor market tightness \( \theta \) and consequently search intensity \( s \) in (3.33).
to changes in the market size. The derived reduced-form matching function not only allows to work with a smaller dimensional model but also helps to relate the model’s conditional elasticities to the observed ones: \( \tilde{\eta}_u \) and \( \tilde{\eta}_v \). Note however that the optimality conditions are obviously unaffected and are still given by \( \omega^w = \eta_u \) and \( \omega^f = \eta_v \). The next proposition states the generalized Hosios-condition in terms of the observed, unconditional elasticities.

**Proposition 3.6.1. Generalized Hosios-condition for unconditional elasticities:** The allocation is efficient if, in addition to \( b = 0 \), the effective bargaining powers coincide with corrected versions of the unconditional elasticities, i.e. if

\[
\omega^w = \tilde{\eta}_u + \frac{\xi}{v} \quad \text{and} \quad \omega^f = \tilde{\eta}_v - \frac{\xi}{v}.
\]  

(3.81)

**Proof.** This follows directly from combining the derived relation of the conditional and unconditional elasticities and proposition 3.4.1. \( \blacksquare \)

Importantly, one must strictly distinguish the unconditional matching function in case of variable search intensity from the case with fixed search effort often used in the literature where optimality is given by \( \omega^w = \tilde{\eta}_u \) instead of (3.81). Observe how the fixed search effort case is nested in this model by letting \( v \to \infty \).

### 3.C Uniqueness and multiplicity of equilibria

This section derives conditions for uniqueness and multiplicity of equilibria for the case of risk-neutrality. I will concentrate on the non-policy case, i.e. \( b = 0 \) and \( T = 0 \) to isolate multiplicity stemming from the matching technology. First, let the implicit solution to the Beveridge curve (3.8) be \( u(\theta) \). If \( \xi = 1 \), then the solution for \( u \) can be expressed explicitly. Take the total differential of the rearranged Beveridge curve, \( uq^w(s(\theta), u, \theta) + u\pi^x = \pi^x \), to arrive at

\[
\begin{align*}
&u \frac{\partial q^w}{\partial s(\theta)} \frac{\partial s(\theta)}{\partial \theta} d\theta + u \frac{\partial q^w}{\partial \theta} d\theta + u \frac{\partial q^w}{\partial u} du + q^w du + \pi^x du = 0 \\
\Leftrightarrow & u \frac{q^w}{s(\theta)} \frac{d}{d\theta} s(\theta) \epsilon^w_\theta + u \frac{q^w}{u} \frac{d}{d\theta} u \epsilon^q_\theta d\theta + u \frac{q^w}{u} \epsilon^q_\theta du + q^w du + \pi^x du = 0 \\
\Leftrightarrow & \frac{q^w}{\theta} \left[ \frac{\eta_u}{v} + \eta_v \right] d\theta + \frac{q^w}{u} \left[ \xi - 1 + \frac{1}{1 - u} \right] du = 0
\end{align*}
\]

Clearly, for all degrees of scale it is always true that \( \epsilon^w_\theta(\theta) < 0 \). For the job search condition (3.33) I work with the simple iso-elastic functional form of the search effort function, \( k(s) = k_0 s^v \), which is also used in the simulation part.
The explicit solution of the job search condition is then

\[ s = \left( \frac{\omega - \frac{c}{v k_0}}{1 - \omega v} \right)^{\frac{1}{\nu}} \theta^\frac{1}{\nu}, \]  

(3.82)

as derived in appendix section 3.B. Therefore the following relations can be established

\[ k(s(\theta)) = \frac{\omega - c}{1 - \omega v}, \]  

(3.83)

\[ k(s(\theta)) - s(\theta)k'(s(\theta)) = (1 - \nu)k(s(\theta)) = \frac{1 - \nu}{v} \frac{\omega}{1 - \omega} c \theta < 0. \]  

(3.84)

Insert the last expression, \( s(\theta) \) and \( u(\theta) \) in the job creation condition (3.14) and rearrange to get

\[ \omega f \left[ \frac{y - h}{r + \pi x} \right] = \frac{c}{q_f(\theta)} + \frac{\omega f}{r + \pi x} \frac{v - 1}{v} - \frac{\omega}{1 - \omega} c \theta. \]  

(3.85)

where \( q_f(\theta) \equiv q_f(s(\theta), u(\theta), \theta) \). Every \( \theta^* \) that solves (3.85) gives an equilibrium.

**Lemma 3.6.1.** \( \frac{dq_f(\theta)}{d\theta} > (\leq) 0 \) is a sufficient (necessary) condition for uniqueness (multiplicity) of equilibria.

**Proof.** Simply observe that \( \frac{dq_f(\theta)}{d\theta} > (\leq) 0 \) is a sufficient (necessary) condition for the right-hand side of (3.85) to monotonically decrease (to be non-monotonic) in \( \theta \).

I will now take a closer look at \( q_f(\theta) \). The elasticity of \( q_f(\theta) \) w.r.t. \( \theta \) is given by

\[ \epsilon_{\theta}^{q_f(\theta)} = \eta_v - 1 + \frac{\eta_u}{v} + (\xi - 1)\epsilon_{\theta}^{u(\theta)}, \]  

(3.86)

where \( \epsilon_{\theta}^{u(\theta)} < 0 \) as derived before. Observe that this elasticity is negative if and only if the following condition holds

\[ \eta - \frac{1 - \nu}{v} + \xi - 1 \left[ 1 + \epsilon_{\theta}^{u(\theta)} \right] < 0. \]  

(3.87)

**Proposition 3.6.2.** Assume that \( |\epsilon_{\theta}^{u(\theta)}| < 1 \) then \( \xi \leq (>) 1 \) is a sufficient (necessary) condition for uniqueness (multiplicity) of equilibria.

**Proof.** This follows directly from equation (3.87).

The assumption of \( |\epsilon_{\theta}^{u(\theta)}| < 1 \) does not seem to be unreasonable. In my simulation, using \( b = T = \sigma = 0 \), this elasticity is \(-0.57\) for the calibration with
$\zeta = 0.5$ and $-0.59$ for $\zeta = 1.5$. Hence, multiple equilibria can only occur with increasing returns to scale of the matching function.

3.D Implementation by wage dependent benefits

Often UB are designed in a way such that they are a constant fraction of the wage, i.e. as a replacement ratio. This section shows how the findings would change in this case. Contrary to absolute UB the optimal replacement ratio should be pro-cyclical (counter-cyclical) if $\zeta > 1$ ($\zeta < 1$). As common in the policy oriented strand of the macro-labor literature I assume that UB are given by $b = \rho w$, where $\rho$ denotes the replacement ratio. While this seems to be an appropriate specification in many contexts it is problematic for the exercise of looking at comparative statics w.r.t. productivity. The reason is that in reality $b$ depends on the previous wage and not on a currently by the business cycle affected wage index as suggested in this specification. Hence, the original specification taking $b$ constant seems to be more appropriate. Nevertheless, I will provide a short analysis of wage dependent benefits. A single bargaining pair takes the wage index as given, i.e. it does not change the sharing rule if it was derived from explicit Nash-bargaining. Hence, one can simply insert $z = h + \rho w - k(s)$ into (3.23) and solve for the wage

$$w = \frac{(1 - \omega) [h - k(s) + sk'(s)] + \omega y}{1 - (1 - \omega)\rho}. \quad (3.88)$$

I have established that optimal benefits in absolute numbers should be proportional to the social surplus and the employment rate. If benefits vary with current wages and wages fluctuate stronger than the match surplus and $1 - u$ than the derived optimal business cycle responses will be reversed. Insert $b = \rho w$ in (3.47) to get

$$\rho = (1 - \zeta) \cdot \frac{\Gamma(1 - u)}{w}. \quad (3.89)$$

Recall that given the assumptions on the functional form of $k(\cdot)$ one can rewrite $k(s) - sk'(s) = (1 - \nu)k(s)$. Insert (3.88) in (3.89) to get

$$\rho = \frac{1 - (1 - \zeta)}{1 - (1 - \omega)\rho} = (1 - \zeta) \cdot \frac{y - h - (\nu - 1)k(s)}{\omega y + (1 - \omega) [h + (\nu - 1)k(s)]}. \quad (3.90)$$

The right-hand side is increasing in $\rho$. It seems that whether or not the right-
hand side increases with $a$ will depend on the bargaining power $\omega$. Clearly, for $\omega \to 1$ the right-hand side is decreasing in $a$ as $ds/da > 0$. For the other case $\omega \to 0$ it is analytically ambiguous but numerical simulations suggest that the relevant term $\frac{y}{h+(\nu-1)k(s)}$ is quite robustly decreasing in $a$ as well. Hence, wages fluctuate stronger than the social surplus and the cyclical pattern of the optimal replacement ratio is reversed compared to UB in absolute terms.

3.E Tables

**Table 3.4: Different calibration scenarios**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\xi$</th>
<th>$\sigma$</th>
<th>$\alpha$</th>
<th>$M_0$</th>
<th>$w$</th>
<th>$s$</th>
<th>$u$</th>
<th>$\theta$</th>
<th>$q^\nu$</th>
<th>$T$</th>
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<tr>
<td>Benchmark</td>
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<td>0.9894</td>
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<td>0.0571</td>
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<td>0.5940</td>
<td>0.0303</td>
</tr>
<tr>
<td>Case 2</td>
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<td>0.0571</td>
<td>0.7188</td>
<td>0.5940</td>
<td>0.0303</td>
</tr>
<tr>
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<td>0.7188</td>
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<tr>
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<tr>
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<td>0.0303</td>
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<td>Case 7</td>
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**Table 3.5: Optimal unemployment benefits for different marginal product assumptions**

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<th>$\alpha = 0.995$</th>
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<td>$\alpha$</td>
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<td>$b$</td>
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<td>0.0437</td>
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<tr>
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<tr>
<td>$u$</td>
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</tr>
<tr>
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</tr>
<tr>
<td>$u$</td>
<td>0.0257</td>
<td>0.0234</td>
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Bibliography


Helpman, E., O. Itskhoki, and S. Redding (2010): “Inequality and unemploy-

Hopenhayn, H. A. and J. P. Nicolini (1997): “Optimal Unemployment Insur-


Hyde, C. E. (1997): “Multiple equilibria in bargaining models of decentralized

International Division of Labor,” *Review of International Economics*, 12, 476–
494.


Kahn, L. (2007): “The Impact of Employment Protection Mandates on Demo-
graphic Temporary Employment Patterns: International Microeconomic Evi-

Kangasharju, A., J. Pehkonen, and S. Pekkala (2005): “Returns to scale in
a matching model: evidence from disaggregated panel data,” *Applied Eco-
nomics*, 37, 115–118.

Papers 5679, National Bureau of Economic Research, Inc.

Keller, G., K. Roberts, and M. Stevens (2010): “Unemployment, Participa-
tion and Market Size,” Mimeo.


Klein, M. W., S. Schuh, and R. K. Triest (2002): “Job creation, job destruction,
and international competition: job flows and trade: the case of NAFTA,”
Working Papers 02-8, Federal Reserve Bank of Boston.


BIBLIOGRAPHY


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